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Hamiltonian structure and noncommutativity in p-brane models with exotic supersymmetry

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Abstract: The hamiltonian of the simplest twistor super p-brane model preserving 3/4 of the $D = 4$ $N = 1$ supersymmetry in the centrally extended symplectic superspace is derived and its symmetries are described. The constraints of the model are covariantly separated into the first- and the second-class sets and the Dirac brackets (D.B.) are constructed. We show the D.B. noncommutativity of the super p-brane coordinates in extended twistor superspace and find the D.B. realization of the OSp(1|8) superalgebra. Established is the coincidence of the D.B. and Poisson bracket realizations of the OSp(1|8) superalgebra on the constraint surface and the absence there of anomaly terms in the commutation relations for the quantized generators of the superalgebra.

Keywords: Superstrings and Heterotic Strings, p-branes, Space-Time Symmetries, Gauge Symmetry.

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Contents

1. Introduction

Recovering underlying symmetries of M-theory attracts a great attention and the generalized holonomy conception [1, 2] extending the approach [3] has been applied to clear up the problem. It resulted in exposing a strong correlation between hidden space-time symmetries and BPS vacuum states permitted by the supersymmetry algebra including tensor central charges (TCC) [4, 5]. Such a correlation implies further generalization of string/brane models [6]–[13] taking into account the TCC presence and associated with BPS states preserving global supersymmetry. A special interest here attract exotic BPS states preserving $\frac{\mathsf{M}-1}{\mathsf{M}}$ fraction and less of $N = 1$ supersymmetry revealed in the $N = 1$ superparticle model [14], as a result of the twistor-like generalization of the superparticle model [15] introducing auxiliary spinor variables [16, 17]. The existence of the BPS states preserving exotic 3/4 fraction of supersymmetry was proved in [18] using a model-independent analysis of $D = 4$, $N = 1$ superalgebra enlarged by the TCC Z_{mn}. The connection of these BPS states with the Jordan algebra of 4×4 real symmetric matrices and associated geometric structures was stressed there. Realizations of supersymmetric configurations with extra supersymmetry in string/M-theory [19], including a general class of pp-wave solutions preserving various fractions of supersymmetry in the type IIB theory and M-theory, were studied in many papers including [20]–[24].

To clear up the dynamical role of TCC coordinates a new class of exactly solvable twistor-like models of (super)string with the action quadratic in derivatives of $D = 4$ space-time and six real TCC coordinates z_{mn} was constructed in [25] as a natural generalization of the twistor-like formulations [26] (see also [27]) for the Nambu-Goto and Schild string actions. Then the set [25] was enlarged by the model of free tensionless super p-branes ($p = 1, 2, \ldots$) [28] with the action linear in derivatives and preserving 1/4, 1/2, 3/4 of the $D = 4$, $N = 1$ supersymmetry enlarged by the TCC Z_{mn}. The model [28] solved the problem of the generalization of the twistor-like superparticle action [14] to superstrings and super p-branes preserving all but one supersymmetries. Moreover, new Wess-Zumino like super p-brane models nonlinear in derivatives and preserving $\frac{\mathsf{M}-1}{\mathsf{M}}$ fraction of supersymmetry were recently proposed in [29]. It was established that the models [29] generate the Dirichlet boundary conditions for the superstrings and superbranes as a consequence of the variational principle.

It was shown in [28] that exotic BPS states preserving all but one of $D = 4$, $N = 1$ supersymmetries are realized by static configurations of free tensionless super p-branes ($p = 1, 2, \ldots$). These static configurations were described by general solutions of the equations of motion of super p-branes evolving in superspace extended by tensor central charge coordinates.[1] Because of the OSp(1|2M) global symmetry of the model in D-dimensional space-time, its static p-brane solution was formulated in terms of supertwistors previously used while studying superparticle models [16, 17, 14] and forming a subspace of the Sp(2M) invariant symplectic space [30, 31]. As a result, the static form of the discussed supertwistor representation of the brane solution is not static in terms of the original superspace-time and TCC coordinates. It is static only modulo transformations of enhanced κ-symmetry and its accompanying local symmetries, since the supertwistor components are invariant under these gauge symmetries, as shown in [32]. Note that invariant character of supertwistors under gauge supersymmetry was previously observed in [17, 33] in the twistorial mechanics of superparticles. The unphysical p-brane motions related to the gauge sym-

[1]The general solutions obtained in [28] have the same form in higher dimensional spaces with the dimension $D = 2, 3, 4 \pmod 8$, where the Majorana spinors exist, and describe exotic BPS states preserving $\frac{\mathsf{M}-1}{\mathsf{M}}$ fraction of $N = 1$ supersymmetry [29].

metries were geometrically realized as the abelian shifts [32] of the space-time and TCC coordinates by the Lorentz vectors and bivectors (generally multivectors) generalizing vector light-like Penrose shifts of the standard space-time coordinates [34]. Being inessential on the classical level of consideration, these shifts may turn out to be essential in the quantum dynamics of strings and branes. This necessitates quantum treatment of the model [28] in the original variables that belong to the superspace extended by TCC coordinates and auxiliary spinor fields.

An interest to this problem is stimulated by the connection [35, 36] of the tensionless strings with free conformal SYM field theory and higher spins based on the relation $g^2_{\rm YM}N = (R^2/\alpha')^2$ between the 't Hooft coupling constant $g^2_{\rm YM}N$ and the rescaled string tension $T = R^2/\alpha'$, where R is the radius of $AdS_5 \times S^5$. This relation was used to conjecture that conformal $\mathcal{N} = 4$ SYM theory has to be dual to massless higher spin field theory (HSFT) [31] when N is large. It was explained in [35, 36] by the observation that at $g^2_{\rm YM}N = 0$, corresponding to the string tension $T = 0$, there is a family of free conformal field theories including conserved tensors of arbitrary spin in the addition to the conserved energy-momentum tensor. Such conformal HSFT depends on N and may be locally formulated in AdS_5 in the $\alpha' \to \infty$ limit.

The model [28] proposes the action for tensionless superstring and super p-branes in the centrally extended superspace just at the critical point $T = 0$ corresponding to $g^2_{\rm YM}N = 0$, where symplectic supersymmetry, playing an important role in the formulation of massless HSFT [30], appears as the symmetry of the model. Note, that the presence of this orthosymplectic symmetry in the massless superparticle model [14] has been a key element to quantize the model and to find the higher spin states in its spectrum [37]. However, the physical situation becomes more complicated in the case of (super)string and (super)branes because, at the point $T = 0$ all massive string excitations from the leading and daughter Regge trajectories are collapsed into the zero mass level. As a result, any state of tensionless string characterized by arbitrary integer spin J is infinitely degenerated and presented by an infinite number of massless states. Then the problem to describe such the states appears and a physical interpretation of the considered model in terms of the phenomenological lagrangian approach [38] with partially broken orthosymplectic global symmetry was proposed in [28, 29]. The point was don't to seek for a description in terms of the microscopic oscillatoric modes of string/brane but, instead to think in terms of the collective macroscopic variables associated with the string coordinates. A crucial example of the phenomenological description was given in [6], where a four-dimensional membrane solution in a six dimensional $N = 1$ supersymmetric gauge theory has been found showing that $D = 4$, $N = 2$ global symmetry of the effective theory is partially broken down to $N = 1$. In behalf of such type of a description in our case is the observation that the general solutions of the model [28] and its higher dimensional generalizations are static branes with frozen oscillatoric modes spontaneously breaking $\mathrm{OSp}(1|2M)$ symmetry. Thus, the generalized coordinates of the model [28] are the Goldstone fields of $\mathrm{OSp}(1|2M)$ symmetry which might give an effective macroscopic description of the vacua of the underlying microscopic HSFT preserving $\frac{M-1}{M}$ fraction of the $N = 1$ supersymmetry. The open problem is to construct this microscopic HSFT.

The Dirac analysis of the hamiltonian structure of tensionless extended objects [39, 40] permits to outline some peculiarities of their quantum dynamics [41, 42, 43]. For the case of dynamical systems including the second-class constraints, such information is accumulated in the Dirac brackets used during the quantization procedure. The brane model [28] contains the first- and the second-class constraints in view of the linear character of the lagrangian in the world-volume time derivatives and the presence of the auxiliary spinor field parametrizing the string/brane momenta. Construction of the Dirac brackets for the discussed super p-brane model implies its hamiltonian analysis with a covariant division of the first- and second-class constraints. We solve this problem here on example of the $D = 4$ $N = 1$ super p-brane model and construct its hamiltonian and Lorentz covariant Dirac brackets. We find that the hamiltonian symplectic structure of the brane model encoded in the Dirac brackets is parametrized by only one dynamical variable ρ^τ describing the proper-time component of the vector world-volume density ρ^μ. The world-volume field ρ^μ plays an important role in the formulation of the super p-brane model [28], because it ensures the world-volume reparametrization and Weyl symmetries of the action. This gauge Weyl symmetry may turn out to be broken on the quantum level signalizing on the anomaly presence similarly to the case of the quantized tensionless string [42].

We start the investigation of the quantum dynamics and find the Dirac bracket (D.B.) noncommutativity between the space-time, TCC and auxiliary brane coordinates. Such a phenomenon for supersymmetric dynamical systems was previously observed in [44, 45] and is under investigation now [46] in field and string theories. To study effects of the noncommutativity on the algebraic level we construct the D.B. realization of the OSp(1|8) superalgebra of the global symmetry of the model. Established is the coincidence between the D.B. and P.B. realizations of the superalgebra, but only on the primary constraint surface. Applying the $\hat{q}\hat{p}$ ordering prescription, we consider a quantum realization of the OSp(1|8) superalgebra and establish that its commutation relations are anomaly free on the constraint surface. However, we note that the total algebra of the brane symmetries which unifies the OSp(1|8) superalgebra with the generalized Virasoro and Weyl world-volume symmetries may include anomalous terms.

2. Lagrangians for strings and branes with enhanced supersymmetry and symplectic twistor

A new simple model [28] describes tensionless strings and p-branes spreading in the symplectic superspace $\mathcal{M}^{\text{susy}}_{\mathsf{M}}$. For $\mathsf{M} = 2^{[\frac{D}{2}]}$ ($D = 2, 3, 4 \mod 8$) this superspace naturally associates with D-dimensional Minkowski space-time extended by the Majorana spinor θ_a ($a = 1, 2, \ldots, 2^{[\frac{D}{2}]}$) and the tensor central charge coordinates z_{ab} additively unified with the standard $x_{ab} = x^m(\gamma_m C^{-1})_{ab}$ space-time coordinates in the symmetric spin-tensor Y_{ab}. The supersymmetric and reparametrization invariant action of the model [28]

$$S_p = \frac{1}{2} \int d\tau d^p\sigma\, \rho^\mu U^a W_{\mu ab} U^b \tag{2.1}$$

includes the world-volume pullback

$$W_{\mu ab} = \partial_\mu Y_{ab} - 2i(\partial_\mu \theta_a \theta_b + \partial_\mu \theta_b \theta_a) \tag{2.2}$$

of the supersymmetric Cartan differential one-form $W_{ab} = W_{\mu ab} d\xi^\mu$, where $\partial_\mu \equiv \frac{\partial}{\partial \xi^\mu}$ and $\xi^\mu = (\tau, \sigma^M)$, $M = 1, 2, \ldots, p$ are world-volume coordinates. The local auxiliary Majorana spinor $U^a(\tau, \sigma^M)$ parametrizes the generalized momentum $P^{ab} = \rho^\tau U^a U^b$ of the tensionless p-brane and $\rho^\mu(\tau, \sigma^M)$ is the world-volume vector density providing the reparametrization invariance of S_p. This action has $(\mathsf{M} - 1)$ κ-symmetries and consequently preserves $\frac{\mathsf{M}-1}{\mathsf{M}}$ fraction of the original global supersymmetry.

By the generalized Penrose transformation of variables

$$Y_{ab} U^b = i\tilde{Y}_a + \tilde{\eta}\theta_a \,, \qquad \tilde{\eta} = -2i(U^a \theta_a) \,, \tag{2.3}$$

where $\tilde{\eta}$ is real Goldstone fermion associated with the spontaneous breakdown of $1/\mathsf{M}$ supersymmetry, the differential one-form $U^a W_{ab} U^b$ is presented as

$$U^a W_{ab} U^b = i\{U^a d\tilde{Y}_a - dU^a \tilde{Y}_a + d\tilde{\eta}\tilde{\eta}\} \equiv dY^\Lambda G_{\Lambda\Sigma} Y^\Sigma \,. \tag{2.4}$$

The new object $Y^\Lambda = (iU^a, \tilde{Y}_a, \tilde{\eta})$ in (2.3), (2.4) is $\mathrm{OSp}(1|2\mathsf{M})$ supertwistor and $G_{\Lambda\Sigma} = (-)^{\Lambda\Sigma+1} G_{\Sigma\Lambda}$ is $\mathrm{OSp}(1|2\mathsf{M})$ invariant supersymplectic metric

$$G_{\Lambda\Sigma} = \begin{pmatrix} \omega^{(2\mathsf{M})} & 0 \\ 0 & i \end{pmatrix} = \begin{pmatrix} 0 & -\delta_a{}^b & 0 \\ \delta^a{}_b & 0 & 0 \\ 0 & 0 & i \end{pmatrix}, \tag{2.5}$$

which is the supersymmetric generalization of $\mathrm{Sp}(2\mathsf{M})$ symplectic metric $\omega^{(2\mathsf{M})}$. In view of (2.3) and (2.4), the action S_p (2.1) is presented in the supertwistor form

$$S_p = \frac{1}{2} \int d\tau d^p\sigma\, \rho^\mu \partial_\mu Y^\Lambda G_{\Lambda\Sigma} Y^\Sigma \tag{2.6}$$

that is apparently invariant under the global generalized superconformal $\mathrm{OSp}(1|2\mathsf{M})$ symmetry. For the particular case of $D = 11$ the action (2.6) invariant under the $\mathrm{OSp}(1|64)$ symmetry was considered in [47].

The original action (2.1) is invariant under $(\mathsf{M} - 1)$ κ-symmetries since the transformation parameter κ_a is restricted by only one real condition

$$U^a \kappa_a = 0 \,, \tag{2.7}$$

as it follows from the transformation rules of the primary variables

$$\delta_\kappa \theta_a = \kappa_a \,, \qquad \delta_\kappa Y_{ab} = -2i(\theta_a \kappa_b + \theta_b \kappa_a) \,, \qquad \delta_\kappa U^a = 0 \,. \tag{2.8}$$

It is easy to show that all components of the supertwistor $Y^\Lambda = (iU^a, \tilde{Y}_a, \tilde{\eta})$ are invariant under κ-symmetry transformations (2.7), (2.8)

$$\delta_\kappa \tilde{Y}_a = 0 \,, \qquad \delta_\kappa \tilde{\eta} = 0 \,, \qquad \delta_\kappa U^a = 0 \,, \tag{2.9}$$

so that the new representation of S_p (2.6) includes only κ-invariant variables. Note that in 4-dimensional space-time Y^Λ contains only 9 real variables that is twice less than the number of the original variables Y_{ab}, θ_a, U^a.

3. Example of the OSp(1|8) invariant string/brane model. Primary constraints

OSp(1|8) is the global supersymmetry of the massless fields of all spins in $D = 4$ space-time extended by TCC coordinates [30, 31]. Therefore, we study $D = 4$ example of the string/brane model (2.1) formulated in generalized $(4 + 6)$-dimensional space $\mathcal{M}_{4+6}$ extended by the Grassmannian Majorana bispinor θ_a. In this case the $D = 4$ $N = 1$ superalgebra

$$\{Q_a, Q_b\} = (\gamma^m C^{-1})_{ab} P_m + i(\gamma^{mn} C^{-1})_{ab} Z_{mn} \tag{3.1}$$

includes the TCC two-form Z_{mn}, and the matrix coordinates Y_{ab} are

$$Y_{ab} = x_{ab} + z_{ab} \,, \tag{3.2}$$

where

$$x_{ab} = x_m (\gamma^m C^{-1})_{ab} \,, \qquad z_{ab} = z_{mn} (\gamma^{mn} C^{-1})_{ab} \tag{3.3}$$

with the charge conjugation matrix C chosen to be imaginary in the Majorana representation. Here we use the same agreements about the spinor algebra as in [28].

In the Weyl basis real symmetric 4×4 matrix Y_{ab} is presented as

$$Y_a{}^b = Y_{ad} C^{db} = \begin{pmatrix} z_\alpha{}^\beta & x_{\alpha\dot\beta} \\ \tilde{x}^{\dot\alpha\beta} & \bar{z}^{\dot\alpha}{}_{\dot\beta} \end{pmatrix} , \tag{3.4}$$

$$C^{ab} = \begin{pmatrix} \epsilon^{\alpha\beta} & 0 \\ 0 & \epsilon_{\dot\alpha\dot\beta} \end{pmatrix} . \tag{3.5}$$

In the $D = 4$ case the auxiliary Majorana spinor $U^a(\tau, \sigma^M)$ together with other auxiliary Majorana spinors $V^a(\tau, \sigma^M)$, $(\gamma_5 U)_a$ and $(\gamma_5 V)_a$ form a local spinor basis

$$U_a = \begin{pmatrix} u_\alpha \\ \bar{u}^{\dot\alpha} \end{pmatrix} , \qquad V_a = \begin{pmatrix} v_\alpha \\ \bar{v}^{\dot\alpha} \end{pmatrix} , \qquad (U\gamma_5 V) = -2i \,, \qquad (UV) = 0 \tag{3.6}$$

attached to string/brane world volume and the γ_5-matrix is

$$(\gamma_5)_a{}^b = \begin{pmatrix} -i\delta_\alpha^\beta & 0 \\ 0 & i\delta_{\dot\beta}^{\dot\alpha} \end{pmatrix} . \tag{3.7}$$

Respectively the linear independent Weyl spinors u^α and v^α may be identified with the local Newman-Penrose dyad [34]

$$u^\alpha v_\alpha \equiv u^\alpha \varepsilon_{\alpha\beta} v^\beta = 1 \,, \qquad u^\alpha u_\alpha = v^\alpha v_\alpha = 0 \,. \tag{3.8}$$

In the Weyl basis the action (2.1) acquires the form

$$S_p = \frac{1}{2} \int d\tau d^p\sigma \, \rho^\mu \left(2u^\alpha \omega_{\mu\alpha\dot\alpha} \bar{u}^{\dot\alpha} + u^\alpha \omega_{\mu\alpha\beta} u^\beta + \bar{u}^{\dot\alpha} \bar\omega_{\mu\dot\alpha\dot\beta} \bar{u}^{\dot\beta} \right) , \tag{3.9}$$

where the supersymmetric one-forms $\omega_{\mu\alpha\dot\alpha}$ and $\omega_{\mu\alpha\beta}$ are

$$\begin{aligned} \omega_{\mu\alpha\dot\alpha} &= \partial_\mu x_{\alpha\dot\alpha} + 2i(\partial_\mu \theta_\alpha \bar\theta_{\dot\alpha} + \partial_\mu \bar\theta_{\dot\alpha} \theta_\alpha) \,, \\ \omega_{\mu\alpha\beta} &= -\partial_\mu z_{\alpha\beta} - 2i(\partial_\mu \theta_\alpha \theta_\beta + \partial_\mu \theta_\beta \theta_\alpha), \\ \bar\omega_{\mu\dot\alpha\dot\beta} &= -\partial_\mu \bar{z}_{\dot\alpha\dot\beta} - 2i(\partial_\mu \bar\theta_{\dot\alpha} \bar\theta_{\dot\beta} + \partial_\mu \bar\theta_{\dot\beta} \bar\theta_{\dot\alpha}) \,. \end{aligned} \tag{3.10}$$

The momenta densities $\mathcal{P}^{\mathfrak{M}}(\tau, \sigma^M)$

$$\mathcal{P}^{\mathfrak{M}} = \frac{\partial L}{\partial \dot{\mathcal{Q}}_{\mathfrak{M}}} = \left(P^{\dot\alpha\alpha}, \pi^{\alpha\beta}, \bar\pi^{\dot\alpha\dot\beta}, \pi^{\alpha}, \bar\pi^{\dot\alpha}, P_u^{\alpha}, \bar P_u^{\dot\alpha}, P_v^{\alpha}, \bar P_v^{\dot\alpha}, P_\mu^{(\rho)}\right) \tag{3.11}$$

are canonically conjugate to the coordinates

$$\mathcal{Q}_{\mathfrak{M}} = (x_{\alpha\dot\alpha}, z_{\alpha\beta}, \bar z_{\dot\alpha\dot\beta}, u_\alpha, \bar u_{\dot\alpha}, v_\alpha, \bar v_{\dot\alpha}, \rho^\mu) \tag{3.12}$$

with respect to the Poisson brackets

$$\{\mathcal{P}^{\mathfrak{M}}(\vec\sigma), \mathcal{Q}_{\mathfrak{N}}(\vec\sigma')\}_{\text{P.B.}} = \delta^{\mathfrak{M}}_{\mathfrak{N}} \delta^p(\vec\sigma - \vec\sigma') \tag{3.13}$$

with the periodic δ-function $\delta^p(\vec\sigma - \vec\sigma')$, where $\vec\sigma = (\sigma^1, \ldots, \sigma^p)$, for the case of closed string/brane studied here.

As far as S_p (3.9) is linear in the proper-time derivatives, it is characterized by the presence of the primary constraints. These constraints may be divided into four sectors.

The bosonic Φ-sector includes the constraints $\Phi \equiv (\Phi^{\dot\alpha\alpha}, \Phi^{\alpha\beta}, \bar\Phi^{\dot\alpha\dot\beta})$ with

$$\begin{aligned}
\Phi^{\dot\alpha\alpha} &= P^{\dot\alpha\alpha} - \rho^\tau u^\alpha \bar u^{\dot\alpha} \approx 0\,, \\
\Phi^{\alpha\beta} &= \pi^{\alpha\beta} + \frac{1}{2}\rho^\tau u^\alpha u^\beta \approx 0\,, \\
\bar\Phi^{\dot\alpha\dot\beta} &= \bar\pi^{\dot\alpha\dot\beta} + \frac{1}{2}\rho^\tau \bar u^{\dot\alpha} \bar u^{\dot\beta} \approx 0\,.
\end{aligned} \tag{3.14}$$

The constraints from the Grassmannian Ψ-sector, where $\Psi = (\Psi^\alpha, \bar\Psi^{\dot\alpha})$, are given by

$$\begin{aligned}
\Psi^\alpha &= \pi^\alpha - 2i\bar\theta_{\dot\alpha} P^{\dot\alpha\alpha} - 4i\pi^{\alpha\beta}\theta_\beta \approx 0\,, \\
\bar\Psi^{\dot\alpha} &= -(\Psi^\alpha)^* = \bar\pi^{\dot\alpha} - 2iP^{\dot\alpha\alpha}\theta_\alpha - 4i\bar\pi^{\dot\alpha\dot\beta}\bar\theta_{\dot\beta} \approx 0\,.
\end{aligned} \tag{3.15}$$

The dyad or (u,v)-sector is formed by the constraints

$$\begin{aligned}
&P_u^\alpha \approx 0\,, \qquad \bar P_u^{\dot\alpha} \approx 0\,, \qquad P_v^\alpha \approx 0\,, \qquad \bar P_v^{\dot\alpha} \approx 0\,, \\
&\Xi \equiv u^\alpha v_\alpha - 1 \approx 0\,, \qquad \bar\Xi \equiv \bar u^{\dot\alpha} \bar v_{\dot\alpha} - 1 \approx 0\,.
\end{aligned} \tag{3.16}$$

Finally, the ρ-sector includes the constraints

$$P_\mu^{(\rho)} \approx 0\,, \qquad \mu = (\tau, M)\,, \qquad M = (1, \ldots, p)\,. \tag{3.17}$$

The constraints forming Φ-sector have zero Poisson brackets (P.B.) among themselves and with the Ψ-sector constraints

$$\{\Phi(\vec\sigma), \Phi(\vec\sigma')\}_{\text{P.B.}} = 0\,, \qquad \{\Phi(\vec\sigma), \Psi(\vec\sigma')\}_{\text{P.B.}} = 0\,. \tag{3.18}$$

They also P.B. commute with P_v^α, $\bar P_v^{\dot\alpha}$, Ξ, $\bar\Xi$ and $P_M^{(\rho)}$

$$\{\Phi(\vec\sigma), P_v^\alpha(\vec\sigma')\}_{\text{P.B.}} = 0\,, \qquad \{\Phi(\vec\sigma), \Xi(\vec\sigma')\}_{\text{P.B.}} = 0\,, \qquad \{\Phi(\vec\sigma), P_M^{(\rho)}(\vec\sigma')\}_{\text{P.B.}} = 0\,. \tag{3.19}$$

The Ψ-constraints have zero P.B. with the constraints from all other sectors, but have nonzero P.B. among themselves. Note that the constraints (3.14)–(3.17) do not contain the space-time $x_{\alpha\dot\alpha}$ and TCC $z_{\alpha\beta}, \bar z_{\dot\alpha\dot\beta}$ coordinates as well as the ρ^M components.

To find all local symmetries of the brane action, it is necessary to split the constraints (3.14)–(3.17) into the first- and second-class constraints. Then the P.B. of the first-class constraints with the brane coordinates will generate the local symmetries in accordance with the Dirac prescription.

4. Ψ-sector: $(3\oplus1)$-splitting and first-class constraints generating enhanced κ-symmetry

Here we find the hamiltonian realization of the enhanced κ-symmetry generators. The generators of κ-symmetry are included into the Ψ-sector (3.15) of the primary constraints due to their Grassmannian graduation. To derive these constraints it is enough to study only Poisson brackets among the Ψ-constraints

$$\{\Psi^{\alpha}(\vec{\sigma}),\Psi^{\beta}(\vec{\sigma}')\}_{\mathrm{P.B.}} = -8i\pi^{\alpha\beta}\delta^{p}(\vec{\sigma}-\vec{\sigma}') = -8i\left(\Phi^{\alpha\beta}-\frac{1}{2}\rho^{\tau}u^{\alpha}u^{\beta}\right)\delta^{p}(\vec{\sigma}-\vec{\sigma}')\,, \quad (4.1)$$

$$\{\Psi^{\alpha}(\vec{\sigma}),\bar{\Psi}^{\dot{\beta}}(\vec{\sigma}')\}_{\mathrm{P.B.}} = -4iP^{\dot{\beta}\alpha}\delta^{p}(\vec{\sigma}-\vec{\sigma}') = -4i\left(\Phi^{\dot{\beta}\alpha}+\rho^{\tau}u^{\alpha}\bar{u}^{\dot{\beta}}\right)\delta^{p}(\vec{\sigma}-\vec{\sigma}')\,, \quad (4.2)$$

because of the commutativity of the Ψ-sector with the others. Then upon the multiplication of (4.1), (4.2) by $u_{\beta}(\tau,\vec{\sigma}')$ and $\bar{u}_{\dot{\beta}}(\tau,\vec{\sigma}')$, respectively, and using (3.8) we find

$$\begin{aligned}\{\Psi^{\alpha}(\vec{\sigma}),\Psi^{(u)}(\vec{\sigma}')\}_{\mathrm{P.B.}} &= -8i\Phi^{\alpha\beta}u_{\beta}\delta^{p}(\vec{\sigma}-\vec{\sigma}')\,,\\ \{\Psi^{\alpha}(\vec{\sigma}),\bar{\Psi}^{(u)}(\vec{\sigma}')\}_{\mathrm{P.B.}} &= -4i\bar{u}_{\dot{\beta}}\Phi^{\dot{\beta}\alpha}\delta^{p}(\vec{\sigma}-\vec{\sigma}')\,.\end{aligned} \quad (4.3)$$

This means that $\Psi^{(u)}$ and $\bar{\Psi}^{(u)}$

$$\Psi^{(u)} \equiv \Psi^{\alpha}u_{\alpha} \approx 0\,, \qquad \bar{\Psi}^{(u)} \equiv \bar{\Psi}^{\dot{\alpha}}\bar{u}_{\dot{\alpha}} \approx 0 \quad (4.4)$$

are the first-class constraints. The Poisson brackets between the $\Psi^{(u)}$, $\bar{\Psi}^{(u)}$ are the following

$$\begin{aligned}\{\Psi^{(u)}(\vec{\sigma}),\bar{\Psi}^{(u)}(\vec{\sigma}')\}_{\mathrm{P.B.}} &= -4i\Phi^{(u)}\delta^{p}(\vec{\sigma}-\vec{\sigma}') \approx 0,\\ \{\Psi^{(u)}(\vec{\sigma}),\Psi^{(u)}(\vec{\sigma}')\}_{\mathrm{P.B.}} &= -8iT^{(u)}\delta^{p}(\vec{\sigma}-\vec{\sigma}') \approx 0,\\ \{\bar{\Psi}^{(u)}(\vec{\sigma}),\bar{\Psi}^{(u)}(\vec{\sigma}')\}_{\mathrm{P.B.}} &= -8i\bar{T}^{(u)}\delta^{p}(\vec{\sigma}-\vec{\sigma}') \approx 0\,,\end{aligned} \quad (4.5)$$

where $T^{(u)}$ and $\Phi^{(u)}$ are the constraints

$$T^{(u)} \equiv u_{\alpha}\Phi^{\alpha\beta}u_{\beta} \approx 0\,, \qquad \bar{T}^{(u)} = (T^{(u)})^{*}\,, \qquad \Phi^{(u)} \equiv \bar{u}_{\dot{\beta}}\Phi^{\dot{\beta}\alpha}u_{\alpha}\,. \quad (4.6)$$

The remaining two constraints from the Ψ-sector are presented by the projections

$$\Psi^{(v)} \equiv \Psi^{\alpha}v_{\alpha}\,, \qquad \bar{\Psi}^{(v)} \equiv \bar{\Psi}^{\dot{\alpha}}\bar{v}_{\dot{\alpha}}\,. \quad (4.7)$$

Projecting the Poisson brackets (4.1), (4.2) on the spinors $v_{\beta}(\tau,\vec{\sigma}')$ and $\bar{v}_{\dot{\beta}}(\tau,\vec{\sigma}')$ and summing up the resulting expressions we find

$$\{\Psi^{\alpha}(\vec{\sigma}),\Psi^{(v)}_{R}(\vec{\sigma}')\}_{\mathrm{P.B.}} = -4i(\bar{v}_{\dot{\beta}}\Phi^{\dot{\beta}\alpha}+2\Phi^{\alpha\beta}v_{\beta})\delta^{p}(\vec{\sigma}-\vec{\sigma}') \approx 0\,, \quad (4.8)$$

where the real constraint $\Psi^{(v)}_{R}$ is defined by the sum

$$\Psi^{(v)}_{R} \equiv \Psi^{(v)}+\bar{\Psi}^{(v)} \approx 0\,. \quad (4.9)$$

The Poisson brackets (4.8) show that $\Psi^{(v)}_{R}$ is the real first-class constraint.

Multiplying (4.8) by $u_\alpha(\tau,\vec{\sigma})$ we find

$$\begin{aligned}\{\Psi^{(u)}(\vec{\sigma}),\Psi_R^{(v)}(\vec{\sigma}')\}_{\text{P.B.}} &= -4i(\bar{v}_{\dot{\beta}}\Phi^{\dot{\beta}\alpha}u_\alpha+2u_\alpha\Phi^{\alpha\beta}v_\beta)\delta^p(\vec{\sigma}-\vec{\sigma}')\approx 0\,,\\ \{\bar{\Psi}^{(u)}(\vec{\sigma}),\Psi_R^{(v)}(\vec{\sigma}')\}_{\text{P.B.}} &= -4i(\bar{u}_{\dot{\beta}}\Phi^{\dot{\beta}\alpha}v_\alpha+2\bar{u}_{\dot{\alpha}}\bar{\Phi}^{\dot{\alpha}\dot{\beta}}\bar{v}_{\dot{\beta}})\delta^p(\vec{\sigma}-\vec{\sigma}')\approx 0\,.\end{aligned}\tag{4.10}$$

The Poisson brackets for $\Psi_R^{(v)}$ with itself are

$$\{\Psi_R^{(v)}(\vec{\sigma}),\Psi_R^{(v)}(\vec{\sigma}')\}_{\text{P.B.}}=-8i\tilde{T}_R^{(v)}\delta^p(\vec{\sigma}-\vec{\sigma}')\approx 0\,,\tag{4.11}$$

where the real constraint $\tilde{T}_R^{(v)}$ from the Φ-sector is defined as

$$\begin{aligned}\tilde{T}_R^{(v)}\equiv T_R^{(v)}+\Phi^{(v)} &= T^{(v)}+\bar{T}^{(v)}+\Phi^{(v)}\approx 0\,,\\ &T^{(v)}\equiv v_\alpha\Phi^{\alpha\beta}v_\beta\,,\qquad \Phi^{(v)}\equiv\bar{v}_{\dot{\alpha}}\Phi^{\dot{\alpha}\alpha}v_\alpha\,.\end{aligned}\tag{4.12}$$

Thus, the Ψ-sector constraints are split into the three real constraints $\Psi^{(u)}$, $\bar{\Psi}^{(u)}$ and $\Psi_R^{(v)}$ belonging to the first-class and one real constraint $\Psi_I^{(v)}$ given by the imaginary part of $\Psi^{(v)}$

$$\Psi_I^{(v)}=i(\Psi^\alpha v_\alpha-\bar{\Psi}^{\dot{\alpha}}\bar{v}_{\dot{\alpha}})\approx 0\,.\tag{4.13}$$

The calculation of the Poisson brackets for $\Psi_I^{(v)}$ with itself yields

$$\{\Psi_I^{(v)}(\vec{\sigma}),\Psi_I^{(v)}(\vec{\sigma}')\}_{\text{P.B.}}=8i(T_R^{(v)}-\Phi^{(v)}-2\rho^\tau)\delta^p(\vec{\sigma}-\vec{\sigma}')\,,\tag{4.14}$$

resulting to the weak equality

$$\{\Psi_I^{(v)}(\vec{\sigma}),\Psi_I^{(v)}(\vec{\sigma}')\}_{\text{P.B.}}\approx -16i\rho^\tau\delta^p(\vec{\sigma}-\vec{\sigma}')\,,\tag{4.15}$$

proving that $\Psi_I^{(v)}$ is the second-class constraint. Therefore, value of the nonzero world-volume field ρ^τ measures breaking of the fourth κ-symmetry of the brane model.

Using the definition of canonical Poisson brackets

$$\{\pi^\alpha(\vec{\sigma}),\theta_\beta(\vec{\sigma}')\}_{\text{P.B.}}=\delta^\alpha_\beta\delta^p(\vec{\sigma}-\vec{\sigma}')\,,\qquad\{\bar{\pi}^{\dot{\alpha}}(\vec{\sigma}),\bar{\theta}_{\dot{\beta}}(\vec{\sigma}')\}_{\text{P.B.}}=\delta^{\dot{\alpha}}_{\dot{\beta}}\delta^p(\vec{\sigma}-\vec{\sigma}')\tag{4.16}$$

we find the transformations of the θ-coordinates under the charges corresponding to the first-class constraints $\Psi^{(u)}$, $\bar{\Psi}^{(u)}$ and $\Psi_R^{(v)}$

$$\begin{aligned}\delta_\kappa\theta_\alpha &= \left\{\int d^p\sigma'\kappa\Psi^{(u)}(\vec{\sigma}'),\theta_\alpha(\vec{\sigma})\right\}_{\text{P.B.}}=\kappa u_\alpha\,,\\ \delta_\kappa\bar{\theta}_{\dot{\alpha}} &= \left\{\int d^p\sigma'\bar{\kappa}\bar{\Psi}^{(u)}(\vec{\sigma}'),\bar{\theta}_{\dot{\alpha}}(\vec{\sigma})\right\}_{\text{P.B.}}=\bar{\kappa}\bar{u}_{\dot{\alpha}}\,,\\ \delta_\kappa x_{\alpha\dot{\alpha}} &= -2i(\kappa u_\alpha\bar{\theta}_{\dot{\alpha}}+\bar{\kappa}\bar{u}_{\dot{\alpha}}\theta_\alpha)\,,\qquad \delta_\kappa z_{\alpha\beta}=-2i\kappa(u_\alpha\theta_\beta+u_\beta\theta_\alpha)\,;\\ \delta_{\kappa_R}\theta_\alpha &= \left\{\int d^p\sigma'\kappa_R\Psi_R^{(v)}(\vec{\sigma}'),\theta_\alpha(\vec{\sigma})\right\}_{\text{P.B.}}=\kappa_R v_\alpha\,,\\ \delta_{\kappa_R}\bar{\theta}_{\dot{\alpha}} &= \left\{\int d^p\sigma'\kappa_R\Psi_R^{(v)}(\vec{\sigma}'),\bar{\theta}_{\dot{\alpha}}(\vec{\sigma})\right\}_{\text{P.B.}}=\kappa_R\bar{v}_{\dot{\alpha}}\,,\\ \delta_{\kappa_R}x_{\alpha\dot{\alpha}} &= -2i\kappa_R(v_\alpha\bar{\theta}_{\dot{\alpha}}+\bar{v}_{\dot{\alpha}}\theta_\alpha)\,,\qquad \delta_{\kappa_R}z_{\alpha\beta}=-2i\kappa_R(v_\alpha\theta_\beta+v_\beta\theta_\alpha)\,.\end{aligned}\tag{4.17}$$

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To connect the transformations (4.17) with the original κ-symmetry transformations (2.8) let us expand κ_α in the dyad basis

$$\kappa_\alpha = -(\kappa_\beta v^\beta)u_\alpha + (\kappa_\beta u^\beta)v_\alpha = -(\kappa_\beta v^\beta)u_\alpha + [\mathrm{Re}(\kappa_\beta u^\beta) + i\,\mathrm{Im}(\kappa_\beta u^\beta)]v_\alpha. \tag{4.18}$$

Then the transformations (2.8) are presented in the form

$$\delta_{\kappa_\alpha}\theta_\alpha = -(\kappa_\beta v^\beta)u_\alpha + \mathrm{Re}(\kappa_\beta u^\beta)v_\alpha, \tag{4.19}$$

in view of the condition $\mathrm{Im}(\kappa_\beta u^\beta) = 0$ equivalent to (2.7). Comparison of (4.17) and (4.19) shows that the transformations (4.17) are the original κ-symmetry transformations with the complex parameter κ and the real parameter κ_R connected with the original parameters κ_α by the relations

$$\kappa = -\kappa_\alpha v^\alpha\,, \qquad \kappa_R = \mathrm{Re}(\kappa_\alpha u^\alpha)\,. \tag{4.20}$$

Therefore, we proved that the first-class constraints $\Psi^{(u)}$, $\bar\Psi^{(u)}$ and $\Psi_R^{(v)}$ are the generators of three κ-symmetries since their Poisson brackets are closed by the constraints from the Φ-sector. We shall comment these Poisson brackets in the next section, where the division of the Φ-sector into the first- and the second-class constraints will be considered.

5. Φ-sector: (6$\oplus$4)-splitting and first-class constraints generating new local symmetries

The Poisson brackets (4.5) of the κ-symmetry constraints $\Psi^{(u)}$ and $\bar\Psi^{(u)}$ are closed by the constraints $\Phi^{(u)}$ and $T^{(u)}$ (4.6) from the Φ-sector. Let us show that the latter constraints are also the first-class ones. It is easy to see that the constraint $\Phi^{(u)}$ transforms the x-coordinates but leaves all other variables in S_p (3.9) intact. The transformation of the x-coordinates is

$$\delta_{\Phi^{(u)}}x_{\alpha\dot\alpha} = \left\{\int d^p\sigma' \epsilon_{\Phi^{(u)}}\Phi^{(u)}(\vec\sigma'), x_{\alpha\dot\alpha}(\vec\sigma)\right\}_{\mathrm{P.B.}} = \epsilon_{\Phi^{(u)}}u_\alpha\bar u_{\dot\alpha} \tag{5.1}$$

and is the local symmetry of the action S_p (3.9)

$$\delta_{\Phi^{(u)}}S_p = \int d\tau d^p\sigma\rho^\mu u^\alpha\partial_\mu(\epsilon_{\Phi^{(u)}}u_\alpha\bar u_{\dot\alpha})\bar u^{\dot\alpha} = 0 \tag{5.2}$$

due to the relation (3.8) $u^\alpha u_\alpha = 0$. Consequently, the constraint $\Phi^{(u)}$ is the first-class one. The transformation (5.1) is the local shift of x_m along the light-like 4-vector $(u\sigma_m\bar u)$

$$\delta_{\Phi^{(u)}}x_m = -\frac{1}{2}\epsilon_{\Phi^{(u)}}(u\sigma_m\bar u) \tag{5.3}$$

and can be rewritten as a weak equality

$$\delta_{\Phi^{(u)}}x_m \approx \tilde\epsilon_{\Phi^{(u)}}P_m\,, \tag{5.4}$$

where $P_m = -2P^{\dot\alpha\alpha}\sigma_{m\alpha\dot\alpha}$ and $\tilde\epsilon_{\Phi^{(u)}} \equiv \epsilon_{\Phi^{(u)}}/\rho^\tau$. On the contrary, the constraints $T^{(u)}$ and $\bar T^{(u)}$ change only the TCC coordinates $z_\alpha{}^\beta$

$$\delta_{T^{(u)}}z_{\alpha\beta} = \epsilon_{T^{(u)}}u_\alpha u_\beta\,, \qquad \delta_{\bar T^{(u)}}\bar z_{\dot\alpha\dot\beta} = \bar\epsilon_{\bar T^{(u)}}\bar u_{\dot\alpha}\bar u_{\dot\beta}, \tag{5.5}$$

as it follows after utilization of the canonical Poisson brackets

$$\{\pi^{\alpha\beta}(\vec{\sigma}), z_{\gamma\delta}(\vec{\sigma}')\}_{\text{P.B.}} = \frac{1}{2}(\delta^{\alpha}_{\gamma}\delta^{\beta}_{\delta} + \delta^{\alpha}_{\delta}\delta^{\beta}_{\gamma})\delta^p(\vec{\sigma} - \vec{\sigma}')\,. \tag{5.6}$$

The transformations (5.5) do not change the action (3.9)

$$\delta_{T^{(u)}} S_p = \frac{1}{2}\int d\tau d^p\sigma\rho^\mu \left[u^\alpha \partial_\mu(\epsilon_{T^{(u)}} u_\alpha u_\beta)u^\beta + \text{c.c.}\right] = 0 \tag{5.7}$$

and, consequently, are the first-class constraints too.

To establish the geometric sense of the transformations (5.5) let us multiply them by $(\sigma_{mn}\varepsilon)^{\alpha\beta}$ and $(\tilde{\sigma}_{mn}\varepsilon)^{\dot{\alpha}\dot{\beta}}$, respectively, and sum up the results. Then we find

$$\delta_{T^{(u)}} z_{mn} = -\frac{i}{4}\left[\epsilon^{(R)}_{T^{(u)}}(u\sigma_{mn}u + \bar{u}\tilde{\sigma}_{mn}\bar{u}) + i\epsilon^{(I)}_{T^{(u)}}(u\sigma_{mn}u - \bar{u}\tilde{\sigma}_{mn}\bar{u})\right] \tag{5.8}$$

using the relation [25]

$$z_{mn} = -\frac{i}{4}\left[z_\alpha{}^\beta \sigma_{mn\beta}{}^\alpha + \bar{z}^{\dot{\beta}}{}_{\dot{\alpha}}\tilde{\sigma}_{mn}{}^{\dot{\alpha}}{}_{\dot{\beta}}\right] \tag{5.9}$$

connecting the spinor representation $z_{\alpha\beta}$ of the TCC coordinates with the tensor representation $z_{mn} = -z_{nm}$. In terms of the Majorana spinor U_a (3.6) the transformation (5.8) is presented as

$$\delta_{T^{(u)}} z_{mn} = \frac{i}{8}\left[\epsilon^{(R)}_{T^{(u)}}(\bar{U}\gamma_{mn}U) + \epsilon^{(I)}_{T^{(u)}}(\bar{U}\gamma_{mn}\gamma_5 U)\right], \tag{5.10}$$

with the real parameters $\epsilon^{(R)}_{T^{(u)}}$ and $\epsilon^{(I)}_{T^{(u)}}$

$$\epsilon^{(R)}_{T^{(u)}} = \frac{1}{2}(\epsilon_{T^{(u)}} + \bar{\epsilon}_{\bar{T}^{(u)}})\,, \qquad \epsilon^{(I)}_{T} = \frac{1}{2i}(\epsilon_{T^{(u)}} - \bar{\epsilon}_{\bar{T}^{(u)}})\,. \tag{5.11}$$

Now let us take into account the observation [26] that the bivectors $(\bar{U}\gamma_{mn}U)$ and $(\bar{U}\gamma_{mn}\gamma_5 U)$ are null tensors, i.e.

$$(\bar{U}\gamma_{mn}U)^2 = 0\,, \qquad (\bar{U}\gamma_{mn}\gamma_5 U)^2 = 0\,. \tag{5.12}$$

This means that $T^{(u)}$ and $\bar{T}^{(u)}$ generate the local shifts of z_{mn} along the isotropic bivectors (5.12) and these shifts are a natural generalization of the vector light-like shift (5.3). On the other hand, these shifts may be presented as the local shifts along the TCC momentum

$$\delta_{T^{(u)}} z_{\alpha\beta} \approx -2\tilde{\epsilon}_{T^{(u)}}\pi_{\alpha\beta}\,, \qquad \delta_{\bar{T}^{(u)}} \bar{z}_{\dot{\alpha}\dot{\beta}} \approx -2\bar{\tilde{\epsilon}}_{\bar{T}^{(u)}}\bar{\pi}_{\dot{\alpha}\dot{\beta}} \tag{5.13}$$

if the primary constraints (3.14) are taken into account. Thus, the Poisson brackets (4.5) can be presented in the form including only the first-class constraints

$$\begin{aligned}
\{\Psi^{(u)}(\vec{\sigma}), \bar{\Psi}^{(u)}(\vec{\sigma}')\}_{\text{P.B.}} &= -8i(uP\bar{u})\delta^p(\vec{\sigma} - \vec{\sigma}')\,, \\
\{\Psi^{(u)}(\vec{\sigma}), \Psi^{(u)}(\vec{\sigma}')\}_{\text{P.B.}} &= -4i(u\pi u)\delta^p(\vec{\sigma} - \vec{\sigma}')\,, \\
\{\bar{\Psi}^{(u)}(\vec{\sigma}), \bar{\Psi}^{(u)}(\vec{\sigma}')\}_{\text{P.B.}} &= -4i(\bar{u}\bar{\pi}\bar{u})\delta^p(\vec{\sigma} - \vec{\sigma}')\,,
\end{aligned} \tag{5.14}$$

where the r.h.s. of (5.14) are the vector $P^{\dot{\alpha}\alpha}$ and the tensor $\pi^{\alpha\beta}$ momenta projections on the isotropic (bi)vectors.

The next first-class constraint from the Φ-sector is the constraint $\tilde{T}^{(v)}_R$ (4.12), which closes the Poisson brackets (4.11) for the extra κ-symmetry generator $\Psi^{(v)}_R$. To prove this observation let us note that $\tilde{T}^{(v)}_R$ transforms only the variables from the Φ-sector $x_{\alpha\dot{\alpha}}$, $z_{\alpha\beta}$, $\bar{z}_{\dot{\alpha}\dot{\beta}}$

$$\delta_{\tilde{T}^{(v)}_R} x_{\alpha\dot{\alpha}} = \epsilon_{\tilde{T}^{(v)}_R} v_\alpha \bar{v}_{\dot{\alpha}} , \qquad \delta_{\tilde{T}^{(v)}_R} z_{\alpha\beta} = \epsilon_{\tilde{T}^{(v)}_R} v_\alpha v_\beta , \qquad \delta_{\tilde{T}^{(v)}_R} \bar{z}_{\dot{\alpha}\dot{\beta}} = \epsilon_{\tilde{T}^{(v)}_R} \bar{v}_{\dot{\alpha}} \bar{v}_{\dot{\beta}} . \tag{5.15}$$

Using the transformations (5.15) we find

$$\delta_{\tilde{T}^{(v)}_R} S_p = \int d\tau d^p\sigma \rho^\mu \partial_\mu \epsilon_{\tilde{T}^{(v)}_R} [(u^\alpha v_\alpha)^2 - 1] = 0 \tag{5.16}$$

and conclude that this transformation is a local symmetry of the brane action (3.9). The symmetry transformation (5.15) describes local shifts of x_m and z_{mn} coordinates along the second light-like direction $(v\sigma_m\bar{v})$ formed by the dyad v_α. But, unlike the light-like shifts (5.3), (5.5), the shifts (5.15) are admissible only due to the mutual cancellation between the x and z contributions into the action variation (5.16).

This result hints that shifts in the directions transversal to the light-like ones $u_\alpha \bar{u}_{\dot{\alpha}}$, $v_\alpha \bar{v}_{\dot{\alpha}}$ and $u_\alpha u_\beta$, $v_\alpha v_\beta$ may be the symmetries of the action (3.9) too. To answer this question let us remind that the real basic orts $m^{(\pm)}_n$ of the local tetrade which are orthogonal to the real light-like orts $n^{(\pm)}_n$ are given by [25, 26]

$$m^{(+)}_{\alpha\dot{\alpha}} = u_\alpha \bar{v}_{\dot{\alpha}} + v_\alpha \bar{u}_{\dot{\alpha}} , \qquad m^{(-)}_{\alpha\dot{\alpha}} = i(u_\alpha \bar{v}_{\dot{\alpha}} - v_\alpha \bar{u}_{\dot{\alpha}}) . \tag{5.17}$$

The local shifts of the x-coordinates in the transverse directions $m^{(\pm)}_{\alpha\dot{\alpha}}$

$$\delta_{\Phi^{(+)}} x_{\alpha\dot{\alpha}} = \epsilon_{\Phi^{(+)}} m^{(+)}_{\alpha\dot{\alpha}} , \qquad \delta_{\Phi^{(-)}} x_{\alpha\dot{\alpha}} = \epsilon_{\Phi^{(-)}} m^{(-)}_{\alpha\dot{\alpha}} \tag{5.18}$$

generated by the constraints $\Phi^{(\pm)}$

$$\Phi^{(+)} \equiv \Phi^{\dot{\alpha}\alpha} m^{(+)}_{\alpha\dot{\alpha}} \approx 0 , \qquad \Phi^{(-)} \equiv \Phi^{\dot{\alpha}\alpha} m^{(-)}_{\alpha\dot{\alpha}} \approx 0 \tag{5.19}$$

change the action (3.9)

$$\begin{aligned} \delta_{\Phi^{(+)}} S_p &= \int d\tau d^p\sigma \rho^\mu \epsilon_{\Phi^{(+)}} (u^\alpha \partial_\mu u_\alpha + \bar{u}^{\dot{\alpha}} \partial_\mu \bar{u}_{\dot{\alpha}}) , \\ \delta_{\Phi^{(-)}} S_p &= i \int d\tau d^p\sigma \rho^\mu \epsilon_{\Phi^{(-)}} (u^\alpha \partial_\mu u_\alpha - \bar{u}^{\dot{\alpha}} \partial_\mu \bar{u}_{\dot{\alpha}}) . \end{aligned} \tag{5.20}$$

However, these variations may be compensated by the corresponding transformations of the TCC coordinates $z_{\alpha\beta}$

$$\begin{aligned} \delta_{T^{(+)}} z_{\alpha\beta} &= \epsilon_{\Phi^{(+)}} u_{\{\alpha} v_{\beta\}} , \qquad & \delta_{T^{(+)}} \bar{z}_{\dot{\alpha}\dot{\beta}} &= \epsilon_{\Phi^{(+)}} \bar{u}_{\{\dot{\alpha}} \bar{v}_{\dot{\beta}\}} ; \\ \delta_{T^{(-)}} z_{\alpha\beta} &= i\epsilon_{\Phi^{(-)}} u_{\{\alpha} v_{\beta\}} , \qquad & \delta_{T^{(-)}} \bar{z}_{\dot{\alpha}\dot{\beta}} &= -i\epsilon_{\Phi^{(-)}} \bar{u}_{\{\dot{\alpha}} \bar{v}_{\dot{\beta}\}} \end{aligned} \tag{5.21}$$

generated by the constraints $T^{(+)}$ and $T^{(-)}$

$$\begin{aligned} T^{(+)} &\equiv \Phi^{\alpha\beta} u_{\{\alpha} v_{\beta\}} + \bar{\Phi}^{\dot{\alpha}\dot{\beta}} \bar{u}_{\{\dot{\alpha}} \bar{v}_{\dot{\beta}\}} = \Phi^{\alpha\beta} u_\alpha v_\beta + \bar{\Phi}^{\dot{\alpha}\dot{\beta}} \bar{u}_{\dot{\alpha}} \bar{v}_{\dot{\beta}} \approx 0 , \\ T^{(-)} &\equiv i[\Phi^{\alpha\beta} u_{\{\alpha} v_{\beta\}} - \bar{\Phi}^{\dot{\alpha}\dot{\beta}} \bar{u}_{\{\dot{\alpha}} \bar{v}_{\dot{\beta}\}}] = i[\Phi^{\alpha\beta} u_\alpha v_\beta - \bar{\Phi}^{\dot{\alpha}\dot{\beta}} \bar{u}_{\dot{\alpha}} \bar{v}_{\dot{\beta}}] \approx 0 , \end{aligned} \tag{5.22}$$

where $u_{\{\alpha}v_{\beta\}} = \frac{1}{2}(u_\alpha v_\beta + u_\beta v_\alpha)$. In fact, the variations of the action (3.9) generated by the doubled constraints $2T^{(+)}$ and $2T^{(-)}$

$$\delta_{2T^{(+)}}S = -\int d\tau d^p\sigma\rho^\mu\epsilon_{\Phi^{(+)}}(u^\alpha\partial_\mu u_\alpha + \bar{u}^{\dot\alpha}\partial_\mu\bar{u}_{\dot\alpha})\,,$$
$$\delta_{2T^{(-)}}S = -i\int d\tau d^p\sigma\rho^\mu\epsilon_{\Phi^{(-)}}(u^\alpha\partial_\mu u_\alpha - \bar{u}^{\dot\alpha}\partial_\mu\bar{u}_{\dot\alpha}) \tag{5.23}$$

exactly compensate the variations (5.20). It proves that the two real constraints $\tilde{T}^{(\pm)}$ belong to the first class

$$\tilde{T}^{(+)} \equiv \Phi^{(+)} + 2T^{(+)} = \Phi^{\dot\alpha\alpha}(u_\alpha\bar{v}_{\dot\alpha} + v_\alpha\bar{u}_{\dot\alpha}) + 2\left(\Phi^{\alpha\beta}u_{\{\alpha}v_{\beta\}} + \bar\Phi^{\dot\alpha\dot\beta}\bar{u}_{\{\dot\alpha}\bar{v}_{\dot\beta\}}\right) \approx 0\,,$$
$$\tilde{T}^{(-)} \equiv \Phi^{(-)} + 2T^{(-)} = i\left[\Phi^{\dot\alpha\alpha}(u_\alpha\bar{v}_{\dot\alpha} - v_\alpha\bar{u}_{\dot\alpha}) + 2\left(\Phi^{\alpha\beta}u_{\{\alpha}v_{\beta\}} - \bar\Phi^{\dot\alpha\dot\beta}\bar{u}_{\{\dot\alpha}\bar{v}_{\dot\beta\}}\right)\right] \approx 0\,. \tag{5.24}$$

Thus, we constructed six real bosonic constraints $\Phi^{(u)}$, $T^{(u)}$, $\bar{T}^{(u)}$ (4.6), $\tilde{T}^{(v)}_R$ (4.12) and $\tilde{T}^{(\pm)}$ (5.24) belonging to the first class out of the ten primary constraints (3.14) of the Φ-sector. Further we find additional first-class constraints that are certain linear combinations of the constraints from the Φ-, (u,v)- and ρ-sectors.

6. Dyad sector: $2\oplus 8$-splitting and first-class constraints

By analogy with the Ψ- and Φ-sectors one can assume that the first-class constraints from the (u,v)-sector describe the local symmetries related to dyad shifts along themselves. The shifts of v_α along u_α

$$\delta_\epsilon v_\alpha = \epsilon u_\alpha\,, \qquad \delta_\epsilon\bar{v}_{\dot\alpha} = \bar\epsilon\bar{u}_{\dot\alpha} \tag{6.1}$$

are generated by the constraints $P^{(u)}_v$ and $\bar{P}^{(u)}_v$

$$P^{(u)}_v \equiv P^\alpha_v u_\alpha \approx 0\,, \qquad \bar{P}^{(u)}_v \equiv \bar{P}^{\dot\alpha}_v\bar{u}_{\dot\alpha} \approx 0\,. \tag{6.2}$$

These shifts supply obvious local symmetry of the action (3.9) since S_p does not depend on v_α. The same is true for the primary constraints from Φ-, Ψ- and ρ-sectors

$$\{P^{(u)}_v(\vec\sigma), \Phi(\vec\sigma')\}_{\rm P.B.} = 0\,, \qquad \{P^{(u)}_v(\vec\sigma), \Psi(\vec\sigma)\}_{\rm P.B.} = 0\,, \qquad \{P^{(u)}_v(\vec\sigma), P^{(\rho)}_\mu(\vec\sigma')\}_{\rm P.B.} = 0\,. \tag{6.3}$$

Moreover, these shifts do not change the Ξ and $\bar\Xi$ constraints that depend on v_α and $\bar{v}_{\dot\alpha}$

$$\{P^{(u)}_v(\vec\sigma), \Xi(\vec\sigma')\}_{\rm P.B.} = 0\,, \qquad \{\bar{P}^{(u)}_v(\vec\sigma), \bar\Xi(\vec\sigma')\}_{\rm P.B.} = 0 \tag{6.4}$$

as it follows from the canonical relations

$$\{P^\alpha_v(\vec\sigma), v_\beta(\vec\sigma')\}_{\rm P.B.} = \delta^\alpha_\beta\delta^p(\vec\sigma - \vec\sigma')\,, \qquad \{\bar{P}^{\dot\alpha}_v(\vec\sigma), \bar{v}_{\dot\beta}(\vec\sigma')\}_{\rm P.B.} = \delta^{\dot\alpha}_{\dot\beta}\delta^p(\vec\sigma - \vec\sigma') \tag{6.5}$$

and their complex conjugate. Therefore, the two real constraints (6.2) are the first-class constraints, and they have zero Poisson brackets between themselves

$$\{P^{(u)}_v(\vec\sigma), P^{(u)}_v(\vec\sigma')\}_{\rm P.B.} = 0\,, \qquad \{P^{(u)}_v(\vec\sigma), \bar{P}^{(u)}_v(\vec\sigma')\}_{\rm P.B.} = 0\,. \tag{6.6}$$

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However, the v-shifts along themselves generated by the constraints $P_v^{(v)}$ and $\bar{P}_v^{(v)}$

$$P_v^{(v)} \equiv P_v^{\alpha} v_{\alpha} \approx 0\,, \qquad \bar{P}_v^{(v)} \equiv \bar{P}_v^{\dot\alpha} \bar{v}_{\dot\alpha} \approx 0 \tag{6.7}$$

do not preserve the constraints Ξ and $\bar{\Xi}$

$$\{P_v^{(v)}(\vec\sigma), \Xi(\vec\sigma')\}_{\mathrm{P.B.}} = (1+\Xi)\delta^p(\vec\sigma-\vec\sigma')\,, \qquad \{\bar{P}_v^{(v)}(\vec\sigma), \bar{\Xi}(\vec\sigma')\}_{\mathrm{P.B.}} = (1+\bar{\Xi})\delta^p(\vec\sigma-\vec\sigma')\,. \tag{6.8}$$

As a result, the constraints (6.7), as well as the constraints $P_u^{(u)}$ and $\bar{P}_u^{(u)}$

$$P_u^{(u)} \equiv P_u^{\alpha} u_{\alpha} \approx 0\,, \qquad \bar{P}_u^{(u)} \equiv \bar{P}_u^{\dot\alpha} \bar{u}_{\dot\alpha} \approx 0 \tag{6.9}$$

are not the first-class ones.

Moreover, the u_α-shifts along v_α, generated by the constraints $P_u^{(v)}$ and $\bar{P}_u^{(v)}$

$$P_u^{(v)} \equiv P_u^{\alpha} v_{\alpha} \approx 0\,, \qquad \bar{P}_u^{(v)} \equiv \bar{P}_u^{\dot\alpha} \bar{v}_{\dot\alpha} \approx 0 \tag{6.10}$$

which also have zero Poisson brackets with Ξ and $\bar{\Xi}$

$$\{P_u^{(v)}(\vec\sigma), \Xi(\vec\sigma')\}_{\mathrm{P.B.}} = 0\,, \qquad \{\bar{P}_u^{(v)}(\vec\sigma), \bar{\Xi}(\vec\sigma')\}_{\mathrm{P.B.}} = 0\,, \tag{6.11}$$

are not the symmetries of S_p. These constraints have also nonzero Poisson brackets with the Φ-sector constraints

$$\begin{aligned} \{\Phi^{\dot\alpha\alpha}(\vec\sigma), P_u^{\beta}(\vec\sigma')\}_{\mathrm{P.B.}} &= \varepsilon^{\alpha\beta}\rho^{\tau}\bar{u}^{\dot\alpha}\delta^p(\vec\sigma-\vec\sigma')\,,\\ \{\Phi^{\alpha\beta}(\vec\sigma), P_u^{\gamma}(\vec\sigma')\}_{\mathrm{P.B.}} &= -\frac{1}{2}\rho^{\tau}(\varepsilon^{\alpha\gamma}u^{\beta} + \varepsilon^{\beta\gamma}u^{\alpha})\delta^p(\vec\sigma-\vec\sigma')\,, \end{aligned} \tag{6.12}$$

where the canonical P.B. relations

$$\{u^{\alpha}(\vec\sigma), P_u^{\beta}(\vec\sigma')\}_{\mathrm{P.B.}} = -\varepsilon^{\alpha\beta}\delta^p(\vec\sigma-\vec\sigma') \tag{6.13}$$

were used. After multiplication of (6.12) by $v_\beta(\tau,\vec\sigma')$ and $v_\gamma(\tau,\vec\sigma')$, respectively, we find

$$\begin{aligned} \{\Phi^{\dot\alpha\alpha}(\vec\sigma), P_u^{(v)}(\vec\sigma')\}_{\mathrm{P.B.}} &= \rho^{\tau}v^{\alpha}\bar{u}^{\dot\alpha}\delta^p(\vec\sigma-\vec\sigma')\,,\\ \{\Phi^{\alpha\beta}(\vec\sigma), P_u^{(v)}(\vec\sigma')\}_{\mathrm{P.B.}} &= -\frac{1}{2}\rho^{\tau}(v^{\alpha}u^{\beta} + v^{\beta}u^{\alpha})\delta^p(\vec\sigma-\vec\sigma')\,, \end{aligned} \tag{6.14}$$

where the r.h.s. of (6.14) do not belong to the primary constraints. So, we resume that the (u,v)-sector itself includes only two real first-class constraints, but, as we shall see below, some of the considered shifts may be compensated by the transformations from the ρ-sector.

7. ρ-sector: $p \oplus 1$-splitting and first-class constraints

The ρ-sector of the p-brane constraints (3.17) includes p constraints of the first-class

$$P_M^{(\rho)} \approx 0\,, \qquad M = (1,\dots,p)\,. \tag{7.1}$$

It follows from the observation that the corresponding canonically conjugate variables ρ^M

$$\{P_M^{(\rho)}(\vec{\sigma}), \rho^N(\vec{\sigma}')\}_{\text{P.B.}} = \delta_M^N \delta^p(\vec{\sigma} - \vec{\sigma}') \tag{7.2}$$

do not enter the primary constraints (3.14)–(3.17) and, consequently, $P_M^{(\rho)}$ have zero Poisson brackets with all these constraints. $P_M^{(\rho)}$ constraints correspond to redefinition of p space components ρ^M of the $(p+1)$-dimensional world-volume vector density $\rho^\mu(\tau, \vec{\sigma})$

$$\delta_\epsilon \rho^M = \epsilon^M(\tau, \vec{\sigma}) \,. \tag{7.3}$$

The transformations (7.3) are new local symmetries of the action S_p due to arbitrariness in the definition of ρ^M. On the other hand, the world-volume time component ρ^τ enters in the Φ-sector constraints and the remaining component $P_\tau^{(\rho)}$ from the ρ-sector has nonzero Poisson brackets with this sector constraints

$$\begin{aligned}
\{P_\tau^{(\rho)}(\vec{\sigma}), \Phi^{\dot{\alpha}\alpha}(\vec{\sigma}')\}_{\text{P.B.}} &= -u^\alpha \bar{u}^{\dot{\alpha}} \delta^p(\vec{\sigma} - \vec{\sigma}') \,, \\
\{P_\tau^{(\rho)}(\vec{\sigma}), \Phi^{\alpha\beta}(\vec{\sigma}')\}_{\text{P.B.}} &= \frac{1}{2} u^\alpha u^\beta \delta^p(\vec{\sigma} - \vec{\sigma}') \,, \\
\{P_\tau^{(\rho)}(\vec{\sigma}), \bar{\Phi}^{\dot{\alpha}\dot{\beta}}(\vec{\sigma}')\}_{\text{P.B.}} &= \frac{1}{2} \bar{u}^{\dot{\alpha}} \bar{u}^{\dot{\beta}} \delta^p(\vec{\sigma} - \vec{\sigma}') \,.
\end{aligned} \tag{7.4}$$

However, the transformation of ρ^τ

$$\delta_\epsilon \rho^\tau = \epsilon^\tau(\tau, \vec{\sigma}) \tag{7.5}$$

generated by the constraint $P_\tau^{(\rho)}$, which is not the first-class one, may be compensated by the corresponding transformation of dyads as we shall show below.

8. The Weyl symmetry constraint

Here we find the first-class constraint showing the presence of the local Weyl symmetry of the brane action. Because the Φ-sector constraints contain the dyad u_α, the transformation (7.5) can be compensated by the following transformation of u_α

$$\delta_\epsilon u_\alpha = -\frac{\epsilon^\tau}{2\rho^\tau} u_\alpha \,, \qquad \delta_\epsilon \bar{u}_{\dot{\alpha}} = -\frac{\epsilon^\tau}{2\rho^\tau} \bar{u}_{\dot{\alpha}} \,, \tag{8.1}$$

generated by the constraint $\Delta^{(u)}$

$$\Delta^{(u)} \equiv P_u^\alpha u_\alpha + \bar{P}_u^{\dot{\alpha}} \bar{u}_{\dot{\alpha}} - 2\rho^\tau P_\tau^{(\rho)} \approx 0 \,. \tag{8.2}$$

However, the new constraint (8.2) has nonzero Poisson brackets with Ξ and $\bar{\Xi}$

$$\{\Delta^{(u)}(\vec{\sigma}), \Xi(\vec{\sigma}')\}_{\text{P.B.}} = (1 + \Xi)\delta^p(\vec{\sigma} - \vec{\sigma}') \,, \qquad \{\Delta^{(u)}(\vec{\sigma}), \bar{\Xi}(\vec{\sigma}')\}_{\text{P.B.}} = (1 + \bar{\Xi})\delta^p(\vec{\sigma} - \vec{\sigma}') \,. \tag{8.3}$$

This noncommutativity can be easily corrected by the compensating transformation of v_α

$$\delta_\epsilon v_\alpha = \frac{\epsilon^\tau}{2\rho^\tau} v_\alpha \,, \qquad \delta_\epsilon \bar{v}_{\dot{\alpha}} = \frac{\epsilon^\tau}{2\rho^\tau} \bar{v}_{\dot{\alpha}} \,. \tag{8.4}$$

It implies the generalization of $\Delta^{(u)}$ (8.2) into the new constraint Δ'_W

$$\Delta'_W \equiv (P^{\alpha}_{u} u_{\alpha} + \bar{P}^{\dot{\alpha}}_{u} \bar{u}_{\dot{\alpha}}) - (P^{\alpha}_{v} v_{\alpha} + \bar{P}^{\dot{\alpha}}_{v} \bar{v}_{\dot{\alpha}}) - 2\rho^{\tau} P^{(\rho)}_{\tau} \approx 0 \tag{8.5}$$

which has zero Poisson brackets with the Φ-sector. Moreover, the Poisson brackets of Δ'_W with the primary constraints (3.14)–(3.17) are equal to zero on the constraint surface. These properties of the constraint Δ'_W will be preserved after addition of the first-class constraints (7.1) which transform Δ'_W to

$$\Delta_W \equiv (P^{\alpha}_{u} u_{\alpha} + \bar{P}^{\dot{\alpha}}_{u} \bar{u}_{\dot{\alpha}}) - (P^{\alpha}_{v} v_{\alpha} + \bar{P}^{\dot{\alpha}}_{v} \bar{v}_{\dot{\alpha}}) - 2\rho^{\mu} P^{(\rho)}_{\mu} \approx 0\,. \tag{8.6}$$

The local transformation generated by Δ_W is the dilation affecting only the auxiliary fields u_{α}, v_{α} and ρ^{μ}

$$\begin{aligned} \rho'^{\mu} &= e^{-2\Lambda}\rho^{\mu}\,, & u'_{\alpha} &= e^{\Lambda} u_{\alpha}\,, & v'_{\alpha} &= e^{-\Lambda} v_{\alpha}\,, \\ x'_{\alpha\dot{\alpha}} &= x_{\alpha\dot{\alpha}}\,, & z'_{\alpha\beta} &= z_{\alpha\beta}\,, & \theta'_{\alpha} &= \theta_{\alpha}\,. \end{aligned} \tag{8.7}$$

The transformation (8.7) is identified with the Weyl symmetry of the p-brane action. From the string point of view, the Weyl invariants $\rho^{\mu} u_{\alpha} \bar{u}_{\dot{\alpha}}$ and $\rho^{\mu} u_{\alpha} u_{\beta}$ constructed from auxiliary world-volume fields are similar to the conventional Weyl invariant of tensile string

$$\sqrt{-g} g^{\mu\nu} \Leftrightarrow \rho^{\mu} u_{\alpha} \bar{u}_{\dot{\alpha}}\,, \tag{8.8}$$

but here (8.7) is the symmetry of the tensionless super p-brane action.

The transformations (7.3) can be used for the gauge fixing

$$\rho^{M} = 0\,. \tag{8.9}$$

Then the Weyl symmetry may be used to fix ρ^{τ} by the gauge condition

$$\dot{\rho}^{\tau}(\tau, \vec{\sigma}) = 0\,, \tag{8.10}$$

or $\rho^{\tau} = \rho_0 = \text{const}$.

Below we shall show that the ρ^{M} translations (7.3) are supplemented by reparametrization transformation of the world-volume coordinates σ_M generated by p first-class constraints formed by intermixing of the primary constraints.

9. Secondary constraints: p space-like reparametrizations

To find the first-class constraints associated with the reparametrizations of p world-volume coordinates σ_M, let us consider the canonical hamiltonian for p-brane action (3.9).

Using the standard definition of the canonical hamiltonian density

$$\begin{aligned} H_0 &= \dot{\mathfrak{Q}}_{\mathfrak{M}} \mathcal{P}^{\mathfrak{M}} - L \\ &= \dot{x}_{\alpha\dot{\alpha}} P^{\dot{\alpha}\alpha} + \dot{z}_{\alpha\beta}\pi^{\alpha\beta} + \dot{\bar{z}}_{\dot{\alpha}\dot{\beta}}\bar{\pi}^{\dot{\alpha}\dot{\beta}} + \dot{\theta}_{\alpha}\pi^{\alpha} + \dot{\bar{\theta}}_{\dot{\alpha}}\bar{\pi}^{\dot{\alpha}} + (\dot{u}_{\alpha} P^{\alpha}_{u} + \dot{v}_{\alpha} P^{\alpha}_{v} + \text{c.c.}) + \dot{\rho}^{\mu} P^{(\rho)}_{\mu} - L \end{aligned} \tag{9.1}$$

and the p-brane lagrangian (3.9) we find

$$H_0 \approx -\rho^{M}\left[u^{\alpha}\omega_{M\alpha\dot{\alpha}}\bar{u}^{\dot{\alpha}} + \frac{1}{2}u^{\alpha}\omega_{M\alpha\beta}u^{\beta} + \frac{1}{2}\bar{u}^{\dot{\alpha}}\bar{\omega}_{M\dot{\alpha}\dot{\beta}}\bar{u}^{\dot{\beta}}\right], \tag{9.2}$$

where we omitted the momenta from the (u,v)- and ρ-sectors that are weakly equal to zero. Taking into account the definitions (3.14) one obtains

$$\begin{aligned} H_0 &\approx -\frac{\rho^M}{\rho^\tau}\left[-(\Phi^{\dot\alpha\alpha}-P^{\dot\alpha\alpha})\omega_{M\alpha\dot\alpha}+(\Phi^{\alpha\beta}-\pi^{\alpha\beta})\omega_{M\alpha\beta}+(\bar\Phi^{\dot\alpha\dot\beta}-\bar\pi^{\dot\alpha\dot\beta})\bar\omega_{M\dot\alpha\dot\beta}\right] \\ &\approx -\frac{\rho^M}{\rho^\tau}\left[P^{\dot\alpha\alpha}\omega_{M\alpha\dot\alpha}-\pi^{\alpha\beta}\omega_{M\alpha\beta}-\bar\pi^{\dot\alpha\dot\beta}\bar\omega_{M\dot\alpha\dot\beta}\right]. \end{aligned} \tag{9.3}$$

Due to the Ψ-constraints (3.15) there is the equality

$$\begin{aligned} l_M^{(\Phi,\Psi)} &\equiv P^{\dot\alpha\alpha}\partial_M x_{\alpha\dot\alpha}+\pi^{\alpha\beta}\partial_M z_{\alpha\beta}+\bar\pi^{\dot\alpha\dot\beta}\partial_M\bar z_{\dot\alpha\dot\beta}+\partial_M\theta_\alpha\pi^\alpha+\partial_M\bar\theta_{\dot\alpha}\bar\pi^{\dot\alpha} \\ &= P^{\dot\alpha\alpha}\omega_{M\alpha\dot\alpha}-\pi^{\alpha\beta}\omega_{M\alpha\beta}-\bar\pi^{\dot\alpha\dot\beta}\bar\omega_{M\dot\alpha\dot\beta}+\partial_M\theta_\alpha\Psi^\alpha+\partial_M\bar\theta_{\dot\alpha}\bar\Psi^{\dot\alpha}, \end{aligned} \tag{9.4}$$

where $l_M^{(\Phi,\Psi)}$ is the reparametrization generator of the Φ- and Ψ-sector coordinates. The total reparametrization generator L_M is connected with $l_M^{(\Phi,\Psi)}$ by the equality

$$L_M = l_M^{(\Phi,\Psi)} + l_M^{(u,v,\rho)}, \tag{9.5}$$

where $l_M^{(u,v,\rho)}$ is the reparametrization generator for the coordinates u, v, ρ

$$l_M^{(u,v,\rho)} \equiv (P_u^\alpha\partial_M u_\alpha + P_v^\alpha\partial_M v_\alpha)+(\bar P_u^{\dot\alpha}\partial_M\bar u_{\dot\alpha}+\bar P_v^{\dot\alpha}\partial_M\bar v_{\dot\alpha})-\partial_M P_\nu^{(\rho)}\rho^\nu \approx 0. \tag{9.6}$$

It follows from eqs. (9.4)–(9.6) that H_0 (9.2) may be presented in the form

$$H_0 \approx -\frac{\rho^M}{\rho^\tau}L_M, \qquad (M=1,\ldots,p). \tag{9.7}$$

As a result of the Dirac selfconsistency condition for the constraints (7.1)

$$\dot P_M^{(\rho)} = \int d^p\sigma'\left\{H_T(\vec\sigma'),P_M^{(\rho)}(\vec\sigma)\right\}_{\mathrm{P.B.}} = \int d^p\sigma'\left\{H_0(\vec\sigma'),P_M^{(\rho)}(\vec\sigma)\right\}_{\mathrm{P.B.}} \approx \frac{1}{\rho^\tau}L_M \approx 0 \tag{9.8}$$

we find p new constraints L_M

$$\begin{aligned} L_M(\mathcal{Q},\mathcal{P}) = {}& P^{\dot\alpha\alpha}\omega_{M\alpha\dot\alpha}-\pi^{\alpha\beta}\omega_{M\alpha\beta}-\bar\pi^{\dot\alpha\dot\beta}\bar\omega_{M\dot\alpha\dot\beta}+\partial_M\theta_\alpha\Psi^\alpha+\partial_M\bar\theta_{\dot\alpha}\bar\Psi^{\dot\alpha}+ \\ &+(P_u^\alpha\partial_M u_\alpha+P_v^\alpha\partial_M v_\alpha)+(\bar P_u^{\dot\alpha}\partial_M\bar u_{\dot\alpha}+\bar P_v^{\dot\alpha}\partial_M\bar v_{\dot\alpha})-\partial_M P_\nu^{(\rho)}\rho^\nu \approx 0. \end{aligned} \tag{9.9}$$

One can check that L_M have Poisson brackets with the primary constraints from all sectors weakly equal to zero. Thus, these constraints realize the σ_M transformations of the reparametrization invariance of the brane action. These secondary constraints complete realization of the Dirac procedure of the first-class constraints construction. The remaining time-like τ-reparametrization constraint is not independent and is constructed from the constraints $\Phi^{(u)}$, $T^{(u)}$ (4.6) for $x_{\alpha\dot\alpha}$, $z_{\alpha\beta}$ and some algebraic combinations of other first-class constraints for the remaining generalized coordinates.

10. Y^Λ supertwistor as an invariant of local symmetries

Here we shall show that $(8_B + 1_F)$ real components of the supertwistor $Y^\Lambda = (iU^a, \tilde{Y}_a, \tilde{\eta})$ in 4-dimensional Minkowski space extended by 6 TCC coordinates are the invariants of $(8_B + 3_F)$ local symmetries generated by the first-class constraints from the Φ-, Ψ- and dyad sectors. We prove that the transition from the original representation of the action (2.1) (or (3.9)) in terms of $(16_B + 4_F)$ variables $(x, z, u, v; \theta)$ to the Y^Λ supertwistor representation (2.6), including $(8_B + 1_F)$ components, preserves all the local symmetries of S_p (2.1). It will follow from the observation that Y^Λ forms a representation of the Weyl symmetry (8.7), the space-like reparametrizations (9.9) and its invariance under the ρ^M-shifts (7.3). All these $(2p+1)$ symmetries remain local symmetries of the supertwistor representation (2.6).

Starting the proof we observe that the supertwistor component $U_a = \binom{u_\alpha}{\bar{u}^{\dot\alpha}}$ is a trivial invariant of all the 11 symmetries generated by the Ψ-, Φ- and (u, v)-sectors. The fermionic component $\tilde{\eta} = -2i(U^a\theta_a) = -2i(u^\alpha\theta_\alpha + \bar{u}_{\dot\alpha}\bar{\theta}^{\dot\alpha})$ is not transformed under symmetry transformations from the Φ- and (u, v)-sectors. Moreover, its invariance under the κ-symmetry generated by $\Psi^{(u)}$, $\bar{\Psi}^{(u)}$ and $\Psi^{(v)}_R$ (4.17) follows from the relations

$$\delta_\kappa\tilde{\eta} = \frac{2}{i}\kappa u^\alpha u_\alpha = 0\,, \qquad \delta_{\bar\kappa}\tilde{\eta} = -\frac{2}{i}\bar{\kappa}\bar{u}^{\dot\alpha}\bar{u}_{\dot\alpha} = 0\,, \qquad \delta_{\tilde{\kappa}_R}\tilde{\eta} = \frac{2}{i}\tilde{\kappa}_R[u^\alpha v_\alpha + \bar{u}_{\dot\alpha}\bar{v}^{\dot\alpha}] \approx 0\,. \tag{10.1}$$

Thus, it remains to prove the invariance of the $\tilde{Y}_a$ components of the supertwistor

$$i\tilde{Y}_a = Y_{ab}U^b - \tilde{\eta}\theta_a = \begin{pmatrix} -z_{\alpha\beta}u^\beta + x_{\alpha\dot\alpha}\bar{u}^{\dot\alpha} - \tilde{\eta}\theta_\alpha \\ \tilde{x}^{\dot\alpha\alpha}u_\alpha - \bar{z}^{\dot\alpha\dot\beta}\bar{u}_{\dot\beta} - \tilde{\eta}\bar{\theta}^{\dot\alpha} \end{pmatrix}. \tag{10.2}$$

Due to the invariance of U_a and $\tilde{\eta}$ variables the variation of $\tilde{Y}_a$ is given by

$$i\delta\tilde{Y}_a = \delta Y_{ab}U^b - \tilde{\eta}\delta\theta_a = \begin{pmatrix} -\delta z_{\alpha\beta}u^\beta + \delta x_{\alpha\dot\alpha}\bar{u}^{\dot\alpha} - \tilde{\eta}\delta\theta_\alpha \\ \delta\tilde{x}^{\dot\alpha\alpha}u_\alpha - \delta\bar{z}^{\dot\alpha\dot\beta}\bar{u}_{\dot\beta} - \tilde{\eta}\delta\bar{\theta}^{\dot\alpha} \end{pmatrix} \tag{10.3}$$

and it is enough to verify the invariance of the upper Weyl component iy_α. Using the relations (4.17) we find that

$$\begin{aligned} i\delta_\kappa y_\alpha &= -2i\kappa u_\alpha\left[u^\beta\theta_\beta - \bar{u}^{\dot\beta}\bar{\theta}_{\dot\beta} - \frac{i}{2}\tilde{\eta}\right] = 0\,, \\ i\delta_{\tilde{\kappa}_R}y_\alpha &= -2i\tilde{\kappa}_R v_\alpha\left[u^\beta\theta_\beta - \bar{u}^{\dot\beta}\bar{\theta}_{\dot\beta} - \frac{i}{2}\tilde{\eta}\right] = 0\,. \end{aligned} \tag{10.4}$$

So, we confirmed the invariant character of $\tilde{Y}_a$ under the κ-symmetry transformations.

The invariance of $\tilde{Y}_a$ under the symmetry transformations (6.1) from the (u, v)-sector is evident, because $\tilde{Y}_a$ does not include v_α.

The transformations of $\tilde{Y}_a$ under the symmetries of the Φ-sector are defined by

$$i\delta\tilde{Y}_a = \begin{pmatrix} -\delta z_{\alpha\beta}u^\beta + \delta x_{\alpha\dot\alpha}\bar{u}^{\dot\alpha} \\ \delta\tilde{x}^{\dot\alpha\alpha}u_\alpha - \delta\bar{z}^{\dot\alpha\dot\beta}\bar{u}_{\dot\beta} \end{pmatrix}, \tag{10.5}$$

because θ_a and $\tilde{\eta}$ are invariants of these symmetries. The invariance of $\tilde{Y}_a$ under the symmetries (5.1) and (5.5) is evident because of the relations

$$\delta_{\Phi(u)} x_{\alpha\dot{\alpha}} \bar{u}^{\dot{\alpha}} = \epsilon_{\Phi(u)} u_\alpha (\bar{u}_{\dot{\alpha}} \bar{u}^{\dot{\alpha}}) = 0, \qquad \delta_{T(u)} z_{\alpha\beta} u^\beta = \epsilon_{T(u)} u_\alpha (u_\beta u^\beta) = 0 \tag{10.6}$$

and their complex conjugate.

The invariance under the symmetry (5.15) follows from the cancellation between the x and z contributions

$$\delta_{\tilde{T}_R^{(v)}} x_{\alpha\dot{\alpha}} \bar{u}^{\dot{\alpha}} - \delta_{\tilde{T}_R^{(v)}} z_{\alpha\beta} u^\beta \approx \epsilon_{\tilde{T}_R^{(v)}} v_\alpha - \epsilon_{\tilde{T}_R^{(v)}} v_\alpha = 0\,. \tag{10.7}$$

Analogous cancellations also take place with respect to the transformations (5.18), (5.21)

$$\begin{aligned} \delta_{\tilde{T}(+)} x_{\alpha\dot{\alpha}} \bar{u}^{\dot{\alpha}} - \delta_{\tilde{T}(+)} z_{\alpha\beta} u^\beta &= \epsilon_{\tilde{T}(+)} \left[m^{(+)}_{\alpha\dot{\alpha}} \bar{u}^{\dot{\alpha}} - u_{\{\alpha} v_{\beta\}} u^\beta \right] \approx 0\,, \\ \delta_{\tilde{T}(-)} x_{\alpha\dot{\alpha}} \bar{u}^{\dot{\alpha}} - \delta_{\tilde{T}(-)} z_{\alpha\beta} u^\beta &= \epsilon_{\tilde{T}(-)} \left[m^{(-)}_{\alpha\dot{\alpha}} \bar{u}^{\dot{\alpha}} - i u_{\{\alpha} v_{\beta\}} u^\beta \right] \approx 0 \end{aligned} \tag{10.8}$$

and their complex conjugate. It completes the proof of invariant character of Y^Λ under $(8_B + 3_F)$ local symmetries generated by the Ψ-, Φ- and (u,v)-sectors. These symmetries show the pure gauge character of $(8_B + 3_F)$ variables among $(16_B + 4_F)$ primary variables $(x, z, u, v; \theta)$. The transition to the supertwistor representation including $(8_B{+}1_F)$ invariant variables Y^Λ just encodes these $(8_B + 3_F)$ degrees of freedom

$$(16_B + 4_F) - (8_B + 3_F) = (8_B + 1_F), \tag{10.9}$$

so we get the next scheme of the reduction of the original brane variables:

$$\begin{aligned} \kappa - \text{symmetry:} \quad & \theta_\alpha, \bar{\theta}_{\dot{\alpha}} \Rightarrow \tilde{\eta}\,, \\ \text{shifts:} \quad & x_{\alpha\dot{\alpha}}, z_{\alpha\beta}, \bar{z}_{\dot{\alpha}\dot{\beta}}; v_\alpha, \bar{v}_{\dot{\alpha}} \Rightarrow (\tilde{Y}_\alpha, \bar{\tilde{Y}}_{\dot{\alpha}})\,. \end{aligned} \tag{10.10}$$

Taking into account the Weyl symmetry transformations (8.7) one can find the Weyl transformation of the supertwistor

$$\acute{Y}^\Sigma = e^{\Lambda(\tau,\vec{\sigma})} Y^\Sigma\,. \tag{10.11}$$

Using this result one verifies that the supertwistor representation (2.6) of S_p is invariant under the Weyl symmetry. Indeed, the transformed action (2.6) is

$$\begin{aligned} S_p' &= \frac{1}{2} \int d\tau d^p\sigma \rho^\mu \left[\partial_\mu Y^\Sigma G_{\Sigma\Xi} Y^\Xi - \partial_\mu \Lambda (Y^\Sigma G_{\Sigma\Xi} Y^\Xi) \right] \\ &= S_p - \frac{1}{2} \int d\tau d^p\sigma \rho^\mu \partial_\mu \Lambda (Y^\Sigma G_{\Sigma\Xi} Y^\Xi), \end{aligned} \tag{10.12}$$

but the last term equals zero, because $Y^\Sigma G_{\Sigma\Xi} Y^\Xi = 0$ and $S_p' = S_p$. Also, the supertwistor action is invariant under p remaining local symmetries (7.3) from ρ-sector and p world-volume reparametrizations generated by the secondary first-class constraints (9.9).

Thus, we conclude that the strong reduction of the number of variables during the transition to the symplectic supertwistor representation (2.6) is a consequence of the described local symmetries. The pure gauge degrees of freedom are eliminated by the change of variables without any gauge fixing.

11. The hamiltonian

Having completed the Dirac prescription of the first-class constraints construction we present the total hamiltonian density of the super p-brane in the form of their linear combination

$$
\begin{aligned}
H_T = {} & \kappa_u \Psi^{(u)} + \bar{\kappa}_u \bar{\Psi}^{(u)} + \kappa_R \Psi_R^{(v)} + a_u \Phi^{(u)} + b_u T^{(u)} + \bar{b}_u \bar{T}^{(u)} + \\
& + c^{(+)} \tilde{T}^{(+)} + c^{(-)} \tilde{T}^{(-)} + c_R^{(v)} \tilde{T}_R^{(v)} + e P_v^{(u)} + \bar{e} \bar{P}_v^{(u)} + \omega \Delta_W + \\
& + f^M P_M^{(\rho)} + \tilde{\rho}^M L_M \approx 0 \,,
\end{aligned}
\tag{11.1}
$$

where the functions κ, a, b, c, e, f, ω and $\tilde{\rho}$ form the set of $(9+2p)_B + 3_F$ real Lagrange multipliers.

In the hamiltonian formalism the second-class constraints are taken into account by the transition to the Dirac brackets (D.B.) which have to be changed by the (anti)commutators in the quantum dynamics. To construct the D.B. we need to find the second-class constraints remaining after the first-class constraint separation. This problem will be solved below.

12. Second-class constraints and the Dirac brackets

To find the second-class constraints we shall follow to the applied above projection method [41, 25, 26]. In the Ψ-sector only one constraint $\Psi_I^{(v)}$ (4.13) has remained which is the second-class constraint, because its P.B.

$$
\left\{ \Psi_I^{(v)}(\vec{\sigma}), \Psi_I^{(v)}(\vec{\sigma}') \right\}_{\text{P.B.}} = 8i[(v_\alpha \Phi^{\alpha\beta} v_\beta + \bar{v}_{\dot{\alpha}} \bar{\Phi}^{\dot{\alpha}\dot{\beta}} \bar{v}_{\dot{\beta}} - \bar{v}_{\dot{\beta}} \Phi^{\dot{\beta}\alpha} v_\alpha) - 2\rho^\tau] \delta^p(\vec{\sigma} - \vec{\sigma}') \tag{12.1}
$$

is equal to

$$
\left\{ \Psi_I^{(v)}(\vec{\sigma}), \Psi_I^{(v)}(\vec{\sigma}') \right\}_{\text{P.B.}} \approx -16 i \rho^\tau \delta^p(\vec{\sigma} - \vec{\sigma}') \,. \tag{12.2}
$$

In the Φ-sector there are four second-class constraints that may be constructed either from the $\Phi^{\alpha\beta}$-subsector alone or from the $\Phi^{\alpha\beta}$- and $\Phi^{\dot{\alpha}\alpha}$-subsectors. We choose the projections of the three remaining constraints from the $\Phi^{\dot{\alpha}\alpha}$-subsector

$$
\begin{aligned}
\Phi & \equiv \Phi^{\dot{\alpha}\alpha}(m^{(+)} - i m^{(-)})_{\alpha\dot{\alpha}} = 2\bar{v}_{\dot{\alpha}} \Phi^{\dot{\alpha}\alpha} u_\alpha \approx 0 \,, \\
\bar{\Phi} & \equiv (\Phi)^* = \Phi^{\dot{\alpha}\alpha}(m^{(+)} + i m^{(-)})_{\alpha\dot{\alpha}} = 2\bar{u}_{\dot{\alpha}} \Phi^{\dot{\alpha}\alpha} v_\alpha \approx 0 \,, \\
\Phi^{(v)} & \equiv \bar{v}_{\dot{\alpha}} \Phi^{\dot{\alpha}\alpha} v_\alpha \approx 0
\end{aligned}
\tag{12.3}
$$

to be the second-class constraints. Then the additional fourth second-class constraint $T_I^{(v)}$ is supplied by the $\Phi^{\alpha\beta}$-subsector

$$
T_I^{(v)} \equiv i(T^{(v)} - \bar{T}^{(v)}) = i(v_\alpha \Phi^{\alpha\beta} v_\beta - \bar{v}_{\dot{\alpha}} \bar{\Phi}^{\dot{\alpha}\dot{\beta}} \bar{v}_{\dot{\beta}}) \approx 0 \,. \tag{12.4}
$$

The second-class constraints (12.3), (12.4) have non-zero P.B. with the constraints belonging to the dyad and ρ-sectors. Therefore, we shall seek for such linear combinations from these sectors that form canonically conjugate pairs with the constraints (12.3), (12.4).

Then we find the following four second-class constraints

$$P_\tau^{(\rho)} \approx 0\,, \qquad \{P_\tau^{(\rho)}(\vec\sigma), \Phi^{(v)}(\vec\sigma')\}_{\text{P.B.}} \approx \delta^p(\vec\sigma - \vec\sigma')\,, \tag{12.5}$$

$$\begin{aligned} P_u^{(v)} \equiv P_u^\alpha v_\alpha \approx 0\,, \qquad & \{P_u^{(v)}(\vec\sigma), \Phi(\vec\sigma')\}_{\text{P.B.}} \approx 2\rho^\tau \delta^p(\vec\sigma - \vec\sigma')\,, \\ \bar P_u^{(v)} \equiv \bar P_u^{\dot\alpha} \bar v_{\dot\alpha} \approx 0\,, \qquad & \{\bar P_u^{(v)}(\vec\sigma), \bar\Phi(\vec\sigma')\}_{\text{P.B.}} \approx 2\rho^\tau \delta^p(\vec\sigma - \vec\sigma') \end{aligned} \tag{12.6}$$

and

$$\begin{aligned} \Delta_I \equiv \Delta_I^{(v)} - \Delta_I^{(u)} &\equiv i(P_v^{(v)} - \bar P_v^{(v)}) - i(P_u^{(u)} - \bar P_u^{(u)}) \\ &\equiv i(P_v^\alpha v_\alpha - \bar P_v^{\dot\alpha} \bar v_{\dot\alpha}) - i(P_u^\alpha u_\alpha - \bar P_u^{\dot\alpha} \bar u_{\dot\alpha}) \approx 0\,, \\ \{\Delta_I(\vec\sigma), T_I^{(v)}(\vec\sigma')\}_{\text{P.B.}} &\approx 2\rho^\tau \delta^p(\vec\sigma - \vec\sigma'). \end{aligned} \tag{12.7}$$

The remaining four second-class constraints forming canonically conjugate pairs are supplied by the constraints belonging to the dyad and ρ-sectors

$$\begin{aligned} \Delta \equiv P_u^{(u)} + P_v^{(v)} - \rho^\tau P_\tau^{(\rho)} - \frac{i}{2}\Delta_I \approx 0\,, \qquad \Xi \approx 0\,, \\ \{\Delta(\vec\sigma), \Xi(\vec\sigma')\}_{\text{P.B.}} \approx 2\delta^p(\vec\sigma - \vec\sigma') \end{aligned} \tag{12.8}$$

and their complex conjugate. As a result, the P.B.'s of 12 bosonic constraints (12.3)–(12.8) and one fermionic constraint (4.13) form the Dirac matrix $\hat{\mathbf{C}}$ having the symplectic form

$$\hat{\mathbf{C}} = \begin{array}{c|ccccccccccccc|} & P_\tau^{(\rho)} & \Phi^{(v)} & P_u^{(v)} & \Phi & \bar P_u^{(v)} & \bar\Phi & \Delta_I & T_I^v & \Delta & \Xi & \bar\Delta & \bar\Xi & \Psi_I^{(v)} \\ \hline P_\tau^{(\rho)} & 0 & -1 & & & & & & & & & & & \\ \Phi^{(v)} & 1 & 0 & & & & & & & & & & & \\ P_u^{(v)} & & & 0 & 2\rho^\tau & & & & & & & & & \\ \Phi & & & -2\rho^\tau & 0 & & & & & & & & & \\ \bar P_u^{(v)} & & & & & 0 & 2\rho^\tau & & & & \mathbf{0} & & & \\ \bar\Phi & & & & & -2\rho^\tau & 0 & & & & & & & \\ \Delta_I & & & & & & & 0 & 2\rho^\tau & & & & & \\ T_I^{(v)} & & & & & & & -2\rho^\tau & 0 & & & & & \\ \Delta & & & & & & & & & 0 & 2 & & & \\ \Xi & & & & \mathbf{0} & & & & & -2 & 0 & & & \\ \bar\Delta & & & & & & & & & & & 0 & 2 & \\ \bar\Xi & & & & & & & & & & & -2 & 0 & \\ \Psi_I^v & & & & & & & & & & & & & -16i\rho^\tau \\ \hline \end{array} \tag{12.9}$$

multiplied by $\delta^p(\vec\sigma - \vec\sigma')$. Then we find the determinant of the matrix $\hat{\mathbf{C}}$

$$\det \hat{\mathbf{C}} = i(4\rho^\tau)^7\,. \tag{12.10}$$

The inverse matrix $\hat{\mathbf{C}^{-1}}$ is used to construct the Dirac brackets

$$\begin{aligned} \{f(\sigma), g(\sigma')\}_{\text{D.B.}} = \{f(\sigma), g(\sigma')\}_{\text{P.B.}} - \\ - \sum \int d^p\sigma'' \{f(\sigma), F(\sigma'')\}_{\text{P.B.}} (\hat{\mathbf{C}}^{-1})^{FG}(\sigma'') \{G(\sigma''), g(\sigma')\}_{\text{P.B.}}\,, \end{aligned} \tag{12.11}$$

where F and G contain the second-class constraint set forming $\hat{\mathbf{C}}$.

The inverse Dirac matrix $\hat{\mathbf{C}}^{-1}$ is given by

$$
\hat{\mathbf{C}}^{-1} =
\begin{array}{c|ccccccccccccc|}
 & P_\tau^{(\rho)} & \Phi^{(v)} & P_u^{(v)} & \Phi & \bar{P}_u^{(v)} & \bar{\Phi} & \Delta_I & T_I^v & \Delta & \Xi & \bar{\Delta} & \bar{\Xi} & \Psi_I^{(v)} \\
\cline{2-14}
P_\tau^{(\rho)} & 0 & 1 & & & & & & & & & & & \\
\Phi^{(v)} & -1 & 0 & & & & & & & & & & & \\
P_u^{(v)} & & & 0 & -\frac{1}{2\rho^\tau} & & & & & & & & & \\
\Phi & & & \frac{1}{2\rho^\tau} & 0 & & & & & & & & & \\
\bar{P}_u^{(v)} & & & & & 0 & -\frac{1}{2\rho^\tau} & & & & \mathbf{0} & & & \\
\bar{\Phi} & & & & & \frac{1}{2\rho^\tau} & 0 & & & & & & & \\
\Delta_I & & & & & & & 0 & -\frac{1}{2\rho^\tau} & & & & & \\
T_I^{(v)} & & & & & & & \frac{1}{2\rho^\tau} & 0 & & & & & \\
\Delta & & & & & & & & & 0 & -\frac{1}{2} & & & \\
\Xi & & & & \mathbf{0} & & & & & \frac{1}{2} & 0 & & & \\
\bar{\Delta} & & & & & & & & & & & 0 & -\frac{1}{2} & \\
\bar{\Xi} & & & & & & & & & & & \frac{1}{2} & 0 & \\
\Psi_I^v & & & & & & & & & & & & & \frac{i}{16\rho^\tau} \\
\cline{2-14}
\end{array}
\qquad (12.12)
$$

multiplied by $\delta^p(\vec{\sigma} - \vec{\sigma}')$.

The matrices $\hat{\mathbf{C}}$ (12.9) and $\hat{\mathbf{C}}^{-1}$ (12.12) define the hamiltonian symplectic structure in the total phase space of the original variables (3.11) and (3.12). It is easy to see that the matrix $\hat{\mathbf{C}}$ can be reduced by the striking out the upper 2×2 submatrix presenting the contribution of the constraints $P_\tau^{(\rho)}$, $\Phi^{(v)}$ strongly equal zero. Indeed, the constraint $\Phi^{(v)} = 0$ can be covariantly solved expressing the coordinate ρ^τ in terms of $(vP\bar{v})$

$$
\rho^\tau = (vP\bar{v})\,. \qquad (12.13)
$$

Thus, the canonical pair $(\rho^\tau, P_\tau^{(\rho)})$ is excluded from $\hat{\mathbf{C}}$ after the substitution of $(vP\bar{v})$ for ρ^τ in the reduced matrix $\hat{\mathbf{C}}_{\text{red}}$. As a result, the partially reduced matrix $\hat{\mathbf{C}}^{-1}_{\text{red}}$ is substituted for $\hat{\mathbf{C}}^{-1}$ in the Dirac brackets (12.11) giving a partially contracted representation for them.

13. Noncommutativity of coordinates under the Dirac brackets

The D.B. (12.11) defines the new commutation relations between the canonical variables encoding the second-class constraint presence.

Analysis of the matrix $\hat{\mathbf{C}}^{-1}$ (12.12) structure shows that the dyad-sector coordinates u_α, v_β commute among themselves

$$
\{u_\alpha(\vec{\sigma}), u_\beta(\vec{\sigma}')\}_{\text{D.B.}} = \{u_\alpha(\vec{\sigma}), v_\beta(\vec{\sigma}')\}_{\text{D.B.}} = \{v_\alpha(\vec{\sigma}), v_\beta(\vec{\sigma}')\}_{\text{D.B.}} = 0 \qquad (13.1)
$$

and with ρ^τ and θ_α

$$
\begin{aligned}
&\{u_\alpha(\vec{\sigma}), \rho^\tau(\vec{\sigma}')\}_{\text{D.B.}} = 0\,, \qquad \{v_\alpha(\vec{\sigma}), \rho^\tau(\vec{\sigma}')\}_{\text{D.B.}} = 0\,, \\
&\{u_\alpha(\vec{\sigma}), \theta_\beta(\vec{\sigma}')\}_{\text{D.B.}} = 0\,, \qquad \{v_\alpha(\vec{\sigma}), \theta_\beta(\vec{\sigma}')\}_{\text{D.B.}} = 0\,.
\end{aligned} \qquad (13.2)
$$

However, they do not commute with $x_{\alpha\dot\alpha}$ and $z_{\alpha\beta}$ coordinates

$$
\begin{aligned}
\{x_{\alpha\dot\alpha}(\vec\sigma), u_\beta(\vec\sigma')\}_{\rm D.B.} &= \frac{1}{\rho^\tau} u_\alpha \bar v_{\dot\alpha} v_\beta \delta^p(\vec\sigma-\vec\sigma')\,,\\
\{x_{\alpha\dot\alpha}(\vec\sigma), v_\gamma(\vec\sigma')\}_{\rm D.B.} &= 0\,,\\
\{z_{\alpha\beta}(\vec\sigma), u_\gamma(\vec\sigma')\}_{\rm D.B.} &= \frac{1}{2\rho^\tau} v_\alpha v_\beta u_\gamma \delta^p(\vec\sigma-\vec\sigma')\,,\\
\{z_{\alpha\beta}(\vec\sigma), v_\gamma(\vec\sigma')\}_{\rm D.B.} &= -\frac{1}{2\rho^\tau} v_\alpha v_\beta v_\gamma \delta^p(\vec\sigma-\vec\sigma')\,.
\end{aligned}
\tag{13.3}
$$

The coordinates θ_α have non-zero D.B.'s among themselves

$$
\begin{aligned}
\{\theta_\alpha(\vec\sigma), \theta_\beta(\vec\sigma')\}_{\rm D.B.} &= \frac{i}{16\rho^\tau} v_\alpha v_\beta \delta^p(\vec\sigma-\vec\sigma')\,,\\
\{\theta_\alpha(\vec\sigma), \bar\theta_{\dot\beta}(\vec\sigma')\}_{\rm D.B.} &= -\frac{i}{16\rho^\tau} v_\alpha \bar v_{\dot\beta} \delta^p(\vec\sigma-\vec\sigma')\,,\\
\{\bar\theta_{\dot\alpha}(\vec\sigma), \bar\theta_{\dot\beta}(\vec\sigma')\}_{\rm D.B.} &= \frac{i}{16\rho^\tau} \bar v_{\dot\alpha} \bar v_{\dot\beta} \delta^p(\vec\sigma-\vec\sigma')
\end{aligned}
\tag{13.4}
$$

and this results in the noncommutativity of the Goldstone fermion $\tilde\eta$ (2.3)

$$
\begin{aligned}
\{\theta_\alpha(\vec\sigma), \tilde\eta(\vec\sigma')\}_{\rm D.B.} &= \frac{1}{4\rho^\tau} v_\alpha \delta^p(\vec\sigma-\vec\sigma')\,,\\
\{\tilde\eta(\vec\sigma), \tilde\eta(\vec\sigma')\}_{\rm D.B.} &= -\frac{i}{\rho^\tau} \delta^p(\vec\sigma-\vec\sigma')\,.
\end{aligned}
\tag{13.5}
$$

But, the projection $(v^\alpha\theta_\alpha)$ associated with the unbroken directions commutes with θ_β and $\tilde\eta$

$$
\{v^\alpha\theta_\alpha(\vec\sigma), \theta_\beta(\vec\sigma')\}_{\rm D.B.} = 0\,, \qquad \{v^\alpha\theta_\alpha(\vec\sigma), \tilde\eta(\vec\sigma')\}_{\rm D.B.} = 0\,.
\tag{13.6}
$$

The coordinates θ_α have nonzero D.B.'s with $x_{\alpha\dot\alpha}$ and $z_{\alpha\beta}$ either:

$$
\begin{aligned}
\{\theta_\alpha(\vec\sigma), x_{\beta\dot\beta}(\vec\sigma')\}_{\rm D.B.} &= \frac{1}{8\rho^\tau} v_\alpha (v_\beta\bar\theta_{\dot\beta} - \theta_\beta \bar v_{\dot\beta}) \delta^p(\vec\sigma-\vec\sigma')\,,\\
\{\theta_\alpha(\vec\sigma), z_{\beta\delta}(\vec\sigma')\}_{\rm D.B.} &= \frac{1}{8\rho^\tau} v_\alpha (\theta_\beta v_\delta + v_\beta\theta_\delta) \delta^p(\vec\sigma-\vec\sigma')\,,\\
\{\theta_\alpha(\vec\sigma), \bar z_{\dot\beta\dot\delta}(\vec\sigma')\}_{\rm D.B.} &= -\frac{1}{8\rho^\tau} v_\alpha (\bar\theta_{\dot\beta}\bar v_{\dot\delta} + \bar v_{\dot\beta}\bar\theta_{\dot\delta}) \delta^p(\vec\sigma-\vec\sigma')\,.
\end{aligned}
\tag{13.7}
$$

However, the projection $(v^\alpha\theta_\alpha)$ has zero D.B.'s with the $x_{\alpha\dot\alpha}$ coordinates

$$
\{v^\alpha\theta_\alpha(\vec\sigma), x_{\beta\dot\beta}(\vec\sigma')\}_{\rm D.B.} = 0\,,
\tag{13.8}
$$

but does not commute with the TCC coordinates $z_{\beta\delta}$

$$
\{v^\alpha\theta_\alpha(\vec\sigma), z_{\beta\delta}(\vec\sigma')\}_{\rm D.B.} = \frac{1}{2\rho^\tau}(v^\alpha\theta_\alpha) v_\beta v_\delta \delta^p(\vec\sigma-\vec\sigma')\,.
\tag{13.9}
$$

The space-time $x_{\alpha\dot\alpha}$ and TCC coordinates $z_{\alpha\beta}$ have nonzero D.B.'s among themselves

$$
\begin{aligned}
\{x_{\alpha\dot\alpha}(\vec\sigma), x_{\beta\dot\beta}(\vec\sigma')\}_{\rm D.B.} &= \frac{i}{4\rho^\tau}(\theta_\alpha\bar v_{\dot\alpha} - v_\alpha\bar\theta_{\dot\alpha})(\theta_\beta\bar v_{\dot\beta} - v_\beta\bar\theta_{\dot\beta}) \delta^p(\vec\sigma-\vec\sigma')\,,\\
\{z_{\alpha\beta}(\vec\sigma), z_{\gamma\delta}(\vec\sigma')\}_{\rm D.B.} &= \frac{i}{4\rho^\tau}(\theta_\alpha v_\beta + \theta_\beta v_\alpha)(\theta_\gamma v_\delta + \theta_\delta v_\gamma) \delta^p(\vec\sigma-\vec\sigma')\,,\\
\{x_{\alpha\dot\alpha}(\vec\sigma), z_{\beta\gamma}(\vec\sigma')\}_{\rm D.B.} &= -\frac{i}{4\rho^\tau}(\theta_\alpha\bar v_{\dot\alpha} - v_\alpha\bar\theta_{\dot\alpha})(\theta_\beta v_\gamma + \theta_\gamma v_\beta) \delta^p(\vec\sigma-\vec\sigma')\,.
\end{aligned}
\tag{13.10}
$$

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The D.B. noncommutativity (13.10) has gauge-dependent character and one can show that the r.h.s. of (13.10) vanishes in the partially fixed κ-symmetry gauge

$$\theta^{(v)} \equiv \theta^{\alpha} v_{\alpha} = 0\,. \tag{13.11}$$

To prove this we use the decomposition of θ_α in the dyad basis

$$\theta_{\alpha} = \theta^{(v)} u_{\alpha} - \theta^{(u)} v_{\alpha} = \theta^{(v)} u_{\alpha} - \left(\mathrm{Re}\,\theta^{(u)} - \frac{i}{4}\tilde{\eta}\right) v_{\alpha} \tag{13.12}$$

and present the multipliers entering the r.h.s. of (13.10) as

$$\begin{aligned}(\theta_{\alpha}\bar{v}_{\dot{\alpha}} - v_{\alpha}\bar{\theta}_{\dot{\alpha}}) &= \frac{i}{2}\tilde{\eta} v_{\alpha}\bar{v}_{\dot{\alpha}} + i\left(\mathrm{Im}\,\theta^{(v)} m^{(+)}_{\alpha\dot{\alpha}} - \mathrm{Re}\,\theta^{(v)} m^{(-)}_{\alpha\dot{\alpha}}\right),\\ (\theta_{\beta} v_{\gamma} + \theta_{\gamma} v_{\beta}) &= \frac{i}{2}\tilde{\eta} v_{\beta} v_{\gamma} - 2\,\mathrm{Re}\,\theta^{(u)} v_{\beta} v_{\gamma} + \theta^{(v)}(u_{\beta} v_{\gamma} + u_{\gamma} v_{\beta})\,.\end{aligned} \tag{13.13}$$

In the gauge (13.11) the representations (13.13) reduce to

$$\begin{aligned}(\theta_{\alpha}\bar{v}_{\dot{\alpha}} - v_{\alpha}\bar{\theta}_{\dot{\alpha}})|_{\text{gauge (13.11)}} &= \frac{i}{2}\tilde{\eta} v_{\alpha}\bar{v}_{\dot{\alpha}}\,,\\ (\theta_{\beta} v_{\gamma} + \theta_{\gamma} v_{\beta})|_{\text{gauge (13.11)}} &= \frac{i}{2}\tilde{\eta} v_{\beta} v_{\gamma}\end{aligned} \tag{13.14}$$

and the substitution of (13.14) into (13.10) yields

$$\begin{aligned}\{x_{\alpha\dot{\alpha}}(\vec{\sigma}), x_{\beta\dot{\beta}}(\vec{\sigma}')\}_{\text{D.B.}}|_{\text{gauge (13.11)}} &= \{z_{\alpha\beta}(\vec{\sigma}), z_{\gamma\delta}(\vec{\sigma}')\}_{\text{D.B.}}|_{\text{gauge (13.11)}} = 0\,,\\ \{x_{\alpha\dot{\alpha}}(\vec{\sigma}), z_{\beta\gamma}(\vec{\sigma}')\}_{\text{D.B.}}|_{\text{gauge (13.11)}} &= 0\,,\end{aligned} \tag{13.15}$$

in view of the relation $\tilde{\eta}^2 = 0$. This proves the gauge-dependent character of the D.B. noncommutativity of the coordinates x and z between themselves.

In contrast to this picture, the D.B. noncommutativity of θ_α with $x_{\alpha\dot{\alpha}}$ and $z_{\alpha\beta}$ coordinates cannot be removed by gauge fixing since

$$\begin{aligned}\{\theta_{\alpha}(\vec{\sigma}), x_{\beta\dot{\beta}}(\vec{\sigma}')\}_{\text{D.B.}}|_{\text{gauge (13.11)}} &= \frac{i\tilde{\eta}}{16\rho^{\tau}} v_{\alpha} v_{\beta}\bar{v}_{\dot{\beta}}\delta^{p}(\vec{\sigma}-\vec{\sigma}')\,,\\ \{\theta_{\alpha}(\vec{\sigma}), z_{\beta\gamma}(\vec{\sigma}')\}_{\text{D.B.}}|_{\text{gauge (13.11)}} &= \frac{i\tilde{\eta}}{16\rho^{\tau}} v_{\alpha} v_{\beta} v_{\gamma}\delta^{p}(\vec{\sigma}-\vec{\sigma}')\,.\end{aligned} \tag{13.16}$$

The same conclusion takes place for the D.B.'s of the light-like projection $(ux\bar{u})$ with the transverse coordinates $x^{(+)}$ and $x^{(-)}$

$$\begin{aligned}\{(ux\bar{u})(\vec{\sigma}), x^{(+)}(\vec{\sigma}')\}_{\text{D.B.}} &= \left(\frac{1}{\rho^{\tau}} x^{(+)} + \frac{\tilde{\eta}}{4}\,\mathrm{Re}\,\theta^{(v)}\right)\delta^{p}(\vec{\sigma}-\vec{\sigma}')\,,\\ \{(ux\bar{u})(\vec{\sigma}), x^{(-)}(\vec{\sigma}')\}_{\text{D.B.}} &= \left(\frac{1}{\rho^{\tau}} x^{(-)} - \frac{\tilde{\eta}}{4}\,\mathrm{Im}\,\theta^{(v)}\right)\delta^{p}(\vec{\sigma}-\vec{\sigma}')\,,\end{aligned} \tag{13.17}$$

whereas

$$\{x^{(+)}(\vec{\sigma}), x^{(-)}(\vec{\sigma}')\}_{\text{D.B.}} = \frac{1}{\rho^{\tau}}\,\mathrm{Re}\,\theta^{(v)}\,\mathrm{Im}\,\theta^{(v)}\delta^{p}(\vec{\sigma}-\vec{\sigma}')\,, \tag{13.18}$$

which vanishes in the gauge (13.11). The D.B.'s (13.17), (13.18) are accompanied with zero D.B.

$$\begin{aligned} \{(vx\bar{v})(\vec{\sigma}), x^{(\pm)}(\vec{\sigma}')\}_{\text{D.B.}} &= 0\,, \\ \{x^{(+)}(\vec{\sigma}), x^{(+)}(\vec{\sigma}')\}_{\text{D.B.}} &= \{x^{(-)}(\vec{\sigma}), x^{(-)}(\vec{\sigma}')\}_{\text{D.B.}} = 0\,, \\ \{(ux\bar{u})(\vec{\sigma}), (ux\bar{u})(\vec{\sigma}')\}_{\text{D.B.}} &= \{(vx\bar{v})(\vec{\sigma}), (vx\bar{v})(\vec{\sigma}')\}_{\text{D.B.}} = 0\,, \\ \{(ux\bar{u})(\vec{\sigma}), (vx\bar{v})(\vec{\sigma}')\}_{\text{D.B.}} &= 0\,. \end{aligned} \tag{13.19}$$

Note also that the $x_{\alpha\dot{\alpha}}$ components have nonzero D.B.'s with $\rho^{\tau}(\tau,\vec{\sigma})$

$$\{x_{\alpha\dot{\alpha}}(\vec{\sigma}), \rho^{\tau}(\vec{\sigma}')\}_{\text{D.B.}} = v_{\alpha}\bar{v}_{\dot{\alpha}}\delta^{p}(\vec{\sigma}-\vec{\sigma}')\,, \tag{13.20}$$

in contrast to the TCC coordinates

$$\{z_{\alpha\beta}(\vec{\sigma}), \rho^{\tau}(\vec{\sigma}')\}_{\text{D.B.}} = 0\,. \tag{13.21}$$

It shows that the classical variable ρ^{τ} has to be changed by the operator projection $(\hat{v}\hat{P}\hat{\bar{v}})$ after quantization in the reduced phase space, as it follows from the constraint (12.13).

In the next section we will study the D.B. and commutator realizations of the superalgebra of OSp(1|8) symmetry which is global symmetry of the brane model.

14. Dirac bracket realization of the OSp(1|8) superalgebra

Having proved the OSp(1|8) symmetry of the super p-brane action (3.9) one can construct its P.B. and D.B. realizations.

The generator densities Q^{α} and $\bar{Q}^{\dot{\alpha}}$ of the $N=1$ global supersymmetry are given by

$$\begin{aligned} Q^{\alpha}(\tau,\vec{\sigma}) &= \pi^{\alpha} + 2i\bar{\theta}_{\dot{\alpha}}P^{\dot{\alpha}\alpha} + 4i\pi^{\alpha\beta}\theta_{\beta}\,, \\ \bar{Q}^{\dot{\alpha}}(\tau,\vec{\sigma}) &= \bar{\pi}^{\dot{\alpha}} + 2iP^{\dot{\alpha}\alpha}\theta_{\alpha} + 4i\bar{\pi}^{\dot{\alpha}\dot{\beta}}\bar{\theta}_{\dot{\beta}} \end{aligned} \tag{14.1}$$

and their P.B.'s have the standard form

$$\begin{aligned} \{Q^{\alpha}(\vec{\sigma}), \bar{Q}^{\dot{\alpha}}(\vec{\sigma}')\}_{\text{P.B.}} &= 4iP^{\dot{\alpha}\alpha}\delta^{p}(\vec{\sigma}-\vec{\sigma}')\,, \\ \{Q^{\alpha}(\vec{\sigma}), Q^{\beta}(\vec{\sigma}')\}_{\text{P.B.}} &= 8i\pi^{\alpha\beta}\delta^{p}(\vec{\sigma}-\vec{\sigma}')\,. \end{aligned} \tag{14.2}$$

The "square roots" S_{γ} and $\bar{S}_{\dot{\gamma}}$ of the generalized conformal boost densities $K_{\gamma\dot{\gamma}}$, $K_{\gamma\lambda}$

$$\begin{aligned} \{S_{\gamma}(\vec{\sigma}), \bar{S}_{\dot{\rho}}(\vec{\sigma}')\}_{\text{P.B.}} &= 4iK_{\gamma\dot{\rho}}\delta^{p}(\vec{\sigma}-\vec{\sigma}')\,, \\ \{S_{\gamma}(\vec{\sigma}), S_{\lambda}(\vec{\sigma}')\}_{\text{P.B.}} &= 4iK_{\gamma\lambda}\delta^{p}(\vec{\sigma}-\vec{\sigma}') \end{aligned} \tag{14.3}$$

are given by

$$\begin{aligned} S_{\gamma}(\tau,\vec{\sigma}) &= z_{\gamma\delta}Q^{\delta} + x_{\gamma\dot{\delta}}\bar{Q}^{\dot{\delta}} - 2i\theta_{\gamma}(\theta_{\delta}\pi^{\delta} + \bar{\theta}_{\dot{\delta}}\bar{\pi}^{\dot{\delta}}) + 4i(u^{\delta}\theta_{\delta} - \bar{u}^{\dot{\delta}}\bar{\theta}_{\dot{\delta}})P_{u\gamma}\,, \\ \bar{S}_{\dot{\gamma}}(\tau,\vec{\sigma}) &= \bar{z}_{\dot{\gamma}\dot{\delta}}\bar{Q}^{\dot{\delta}} + x_{\delta\dot{\gamma}}Q^{\delta} - 2i\bar{\theta}_{\dot{\gamma}}(\theta_{\delta}\pi^{\delta} + \bar{\theta}_{\dot{\delta}}\bar{\pi}^{\dot{\delta}}) - 4i(u^{\delta}\theta_{\delta} - \bar{u}^{\dot{\delta}}\bar{\theta}_{\dot{\delta}})\bar{P}_{u\dot{\gamma}}\,. \end{aligned} \tag{14.4}$$

The generalized conformal boost densities $K_{\gamma\lambda}$, $K_{\gamma\dot{\gamma}}$ are respectively presented as

$$\begin{aligned}
K_{\gamma\lambda}(\tau,\vec{\sigma}) &= 2z_{\gamma\beta}z_{\lambda\delta}\pi^{\beta\delta} + 2x_{\gamma\dot{\beta}}x_{\lambda\dot{\delta}}\bar{\pi}^{\dot{\beta}\dot{\delta}} + z_{\gamma\beta}x_{\lambda\dot{\delta}}P^{\dot{\delta}\beta} + x_{\gamma\dot{\beta}}z_{\lambda\delta}P^{\dot{\beta}\delta} + \\
&+ \theta_\lambda(z_{\gamma\delta}\pi^{\delta} + x_{\gamma\dot{\delta}}\bar{\pi}^{\dot{\delta}}) + \theta_\gamma(z_{\lambda\delta}\pi^{\delta} + x_{\lambda\dot{\delta}}\bar{\pi}^{\dot{\delta}}) + \\
&+ (u^{\delta}z_{\delta\lambda} - \bar{u}^{\dot{\delta}}x_{\lambda\dot{\delta}})P_{u\gamma} + (u^{\delta}z_{\delta\gamma} - \bar{u}^{\dot{\delta}}x_{\gamma\dot{\delta}})P_{u\lambda} - \\
&- 2i(u^{\delta}\theta_\delta - \bar{u}^{\dot{\delta}}\bar{\theta}_{\dot{\delta}})(\theta_\lambda P_{u\gamma} + \theta_\gamma P_{u\lambda})\,, \qquad (14.5)\\
K_{\gamma\dot{\gamma}}(\tau,\vec{\sigma}) &= z_{\gamma\delta}\bar{z}_{\dot{\gamma}\dot{\delta}}P^{\dot{\delta}\delta} + x_{\gamma\dot{\delta}}x_{\delta\dot{\gamma}}P^{\dot{\delta}\delta} + 2(z_{\gamma\delta}x_{\lambda\dot{\gamma}}\pi^{\delta\lambda} + x_{\gamma\dot{\delta}}\bar{z}_{\dot{\gamma}\dot{\lambda}}\bar{\pi}^{\dot{\delta}\dot{\lambda}}) + \\
&+ \theta_\gamma(\bar{z}_{\dot{\gamma}\dot{\delta}}\bar{\pi}^{\dot{\delta}} + x_{\delta\dot{\gamma}}\pi^{\delta}) + \bar{\theta}_{\dot{\gamma}}(z_{\gamma\delta}\pi^{\delta} + x_{\gamma\dot{\delta}}\bar{\pi}^{\dot{\delta}}) + \\
&+ (u^{\delta}x_{\delta\dot{\gamma}} - \bar{u}^{\dot{\delta}}\bar{z}_{\dot{\delta}\dot{\gamma}})P_{u\gamma} + (x_{\gamma\dot{\delta}}\bar{u}^{\dot{\delta}} - z_{\gamma\delta}u^{\delta})\bar{P}_{u\dot{\gamma}} - \\
&- 2i(u^{\delta}\theta_\delta - \bar{u}^{\dot{\delta}}\bar{\theta}_{\dot{\delta}})(\bar{\theta}_{\dot{\gamma}}P_{u\gamma} - \theta_\gamma\bar{P}_{u\dot{\gamma}})\,. \qquad (14.6)
\end{aligned}$$

Using the matrix multiplication agreement one can present (14.5) and (14.6) in more compact form

$$\begin{aligned}
K_{\gamma\lambda}(\tau,\vec{\sigma}) &= 2(z\pi z)_{\gamma\lambda} + 2(x\bar{\pi}x)_{\gamma\lambda} + (xPz)_{\gamma\lambda} + (xPz)_{\lambda\gamma} - \\
&- [(z\pi)_\gamma\theta_\lambda + (z\pi)_\lambda\theta_\gamma] - [(x\bar{\pi})_\gamma\theta_\lambda + (x\bar{\pi})_\lambda\theta_\gamma] + \\
&+ [P_{u\gamma}(zu)_\lambda + P_{u\lambda}(zu)_\gamma] - [P_{u\gamma}(x\bar{u})_\lambda + P_{u\lambda}(x\bar{u})_\gamma] - \\
&- 2i((u\theta) - (\bar{u}\bar{\theta}))(\theta_\gamma P_{u\lambda} + \theta_\lambda P_{u\gamma})\,, \qquad (14.7)
\end{aligned}$$

and respectively

$$\begin{aligned}
K_{\gamma\dot{\gamma}}(\tau,\vec{\sigma}) &= (\bar{z}Pz)_{\dot{\gamma}\gamma} + (xPx)_{\gamma\dot{\gamma}} + 2(z\pi x + x\bar{\pi}\bar{z})_{\gamma\dot{\gamma}} + \\
&+ \theta_\gamma(\pi x + \bar{z}\bar{\pi})_{\dot{\gamma}} + \bar{\theta}_{\dot{\gamma}}(z\pi + x\bar{\pi})_\gamma + \\
&+ P_{u\gamma}(ux - \bar{u}\bar{z})_{\dot{\gamma}} + (x\bar{u} - zu)_\gamma\bar{P}_{u\dot{\gamma}} - \\
&- 2i((u\theta) - (\bar{u}\bar{\theta}))(\bar{\theta}_{\dot{\gamma}}P_{u\gamma} - \theta_\gamma\bar{P}_{u\dot{\gamma}})\,. \qquad (14.8)
\end{aligned}$$

The remaining 16 generator densities of the Sp(8) algebra $L_{\alpha\beta}$, $L_{\gamma\dot{\rho}}$ (together with their complex conjugate) are defined by the P.B.'s

$$\{Q^{\gamma}(\vec{\sigma}), S_\rho(\vec{\sigma}')\}_{\text{P.B.}} = 4iL^{\gamma}{}_{\rho}\delta^p(\vec{\sigma}-\vec{\sigma}')\,, \qquad \{Q^{\gamma}(\vec{\sigma}), \bar{S}_{\dot{\rho}}(\vec{\sigma}')\}_{\text{P.B.}} = 4iL^{\gamma}{}_{\dot{\rho}}\delta^p(\vec{\sigma}-\vec{\sigma}')\,, \quad (14.9)$$

and have the form

$$\begin{aligned}
L^{\alpha}{}_{\beta}(\tau,\vec{\sigma}) &= P^{\dot{\beta}\alpha}x_{\beta\dot{\beta}} + 2\pi^{\alpha\gamma}z_{\gamma\beta} - \pi^{\alpha}\theta_\beta + u^{\alpha}P_{u\beta}\,, \\
L^{\alpha}{}_{\dot{\beta}}(\tau,\vec{\sigma}) &= 2\pi^{\alpha\gamma}x_{\gamma\dot{\beta}} + P^{\dot{\gamma}\alpha}\bar{z}_{\dot{\beta}\dot{\gamma}} - \pi^{\alpha}\bar{\theta}_{\dot{\beta}} - u^{\alpha}\bar{P}_{u\dot{\beta}} \qquad (14.10)
\end{aligned}$$

completed by their complex conjugate P.B.'s. The derived P.B. realization of the OSp(1|8) superalgebra together with the definition of the Dirac brackets (12.11) are enough for construction of the D.B. realization of the OSp(1|8) superalgebra.

Keeping in mind the results [42] one can wait the anomaly presence in the commutator of the operator densities $K_{\gamma\dot{\gamma}}$ and $L^{\alpha}{}_{\beta}$. To this end let us in the first place calculate the

Dirac bracket of the generalized superconformal boost density $K_{\gamma\dot\gamma}$ with $L^{\alpha}{}_{\beta}$. Then we find

$$\begin{aligned}\{K_{\gamma\dot\gamma}(\vec\sigma), L^{\alpha}{}_{\beta}(\vec\sigma')\}_{\text{D.B.}} = {} & -\delta^{\alpha}_{\gamma}K_{\beta\dot\gamma}\delta^{p}(\vec\sigma-\vec\sigma')- \\ & -\int d^p\sigma''\{K_{\gamma\dot\gamma}(\vec\sigma),\Phi(\vec\sigma'')\}_{\text{P.B.}}\left(\frac{1}{2\rho^{\tau}(\vec\sigma'')}\right)\{P^{(v)}_{u}(\vec\sigma''), L^{\alpha}{}_{\beta}(\vec\sigma')\}_{\text{P.B.}}+ \\ & +\int d^p\sigma''\{K_{\gamma\dot\gamma}(\vec\sigma),\bar P^{(v)}_{u}(\vec\sigma'')\}_{\text{P.B.}}\left(\frac{1}{2\rho^{\tau}(\vec\sigma'')}\right)\{\bar\Phi(\vec\sigma''), L^{\alpha}{}_{\beta}(\vec\sigma')\}_{\text{P.B.}}+ \\ & +\int d^p\sigma''\{K_{\gamma\dot\gamma}(\vec\sigma),\Delta_I(\vec\sigma'')\}_{\text{P.B.}}\left(\frac{1}{2\rho^{\tau}(\vec\sigma'')}\right)\{T^{(v)}_{I}(\vec\sigma''), L^{\alpha}{}_{\beta}(\vec\sigma')\}_{\text{P.B.}}- \\ & -\int d^p\sigma''\{K_{\gamma\dot\gamma}(\vec\sigma),\Psi^{(v)}_{I}(\vec\sigma'')\}_{\text{P.B.}}\left(\frac{i}{16\rho^{\tau}(\vec\sigma'')}\right)\{\Psi^{(v)}_{I}(\vec\sigma''), L^{\alpha}{}_{\beta}(\vec\sigma')\}_{\text{P.B.}}\end{aligned} \tag{14.11}$$

with the omitted terms vanishing in the strong sense. The terms in the r.h.s. of the D.B. (14.11) containing the non-local factor $1/\rho^{\tau}$ have a special structure in view of which each of them is proportional to some of the constraints of the model. As a result, we find that the contribution of the factor $1/\rho^{\tau}$ will be vanishing on the constraint surface.

To prove this fact we note that the second term in (14.11) includes the two multipliers

$$\begin{aligned}\{K_{\gamma\dot\gamma}(\vec\sigma), \Phi(\vec\sigma'')\}_{\text{P.B.}} &= 2\Big[\rho^{\tau}u_{\gamma}u^{\beta}(x_{\beta\dot\gamma}-2i\theta_{\beta}\bar\theta_{\dot\gamma}) - \rho^{\tau}u_{\gamma}\bar u^{\dot\beta}(\bar z_{\dot\beta\dot\gamma}-2i\bar\theta_{\dot\beta}\bar\theta_{\dot\gamma}) - \\ &\qquad - u_{\gamma}\bar v_{\dot\delta}\bar L^{\dot\delta}{}_{\dot\gamma} - \bar v_{\dot\gamma}u_{\delta}L^{\delta}{}_{\gamma}\Big]\delta^{p}(\vec\sigma-\vec\sigma''), \\ \{P^{(v)}_{u}(\vec\sigma''), L^{\alpha}{}_{\beta}(\vec\sigma')\}_{\text{P.B.}} &= v^{\alpha}P_{u\beta}\delta^{p}(\vec\sigma''-\vec\sigma'),\end{aligned} \tag{14.12}$$

where $\bar L^{\dot\delta}{}_{\dot\gamma}$ is the Lorentz generator complex conjugate to $L^{\delta}{}_{\gamma}$. One can see that the third multiplier is vanished, because of the primary constraint $P_{u\beta}\approx 0$ presence.

The third term in (14.11) also includes the multipliers equal

$$\begin{aligned}\{K_{\gamma\dot\gamma}(\vec\sigma), \bar P^{(v)}_{u}(\vec\sigma'')\}_{\text{P.B.}} &= \Big[\bar v^{\dot\delta}(\bar z_{\dot\delta\dot\gamma}-2i\bar\theta_{\dot\delta}\bar\theta_{\dot\gamma})P_{u\gamma} - \bar v^{\dot\delta}(x_{\gamma\dot\delta}-2i\bar\theta_{\dot\delta}\theta_{\gamma})\bar P_{u\dot\gamma}\Big]\delta^{p}(\vec\sigma-\vec\sigma''), \\ \{\bar\Phi(\vec\sigma''), L^{\alpha}{}_{\beta}(\vec\sigma')\}_{\text{P.B.}} &= 2v_{\beta}\bar u_{\dot\lambda}P^{\dot\lambda\alpha}\delta^{p}(\vec\sigma''-\vec\sigma')\end{aligned} \tag{14.13}$$

and the former is proportional to the primary constraint $P_{u\gamma}\approx 0$ and it complex conjugate. The same story is repeated for the fourth term in (14.11) which multipliers are equal to

$$\begin{aligned}\{K_{\gamma\dot\gamma}(\vec\sigma), \Delta_I(\vec\sigma'')\}_{\text{P.B.}} &= 2i\Big[\bar u^{\dot\delta}(\bar z_{\dot\delta\dot\gamma}-2i\bar\theta_{\dot\delta}\bar\theta_{\dot\gamma})P_{u\gamma} - u^{\delta}(z_{\delta\gamma}-2i\theta_{\delta}\theta_{\gamma})\bar P_{u\dot\gamma}\Big]\delta^{p}(\vec\sigma-\vec\sigma''), \\ \{T^{(v)}_{I}(\vec\sigma''), L^{\alpha}{}_{\beta}(\vec\sigma')\}_{\text{P.B.}} &= 2iv_{\beta}v_{\lambda}\pi^{\lambda\alpha}\delta^{p}(\vec\sigma''-\vec\sigma')\end{aligned} \tag{14.14}$$

and the first of them is vanished, because of the constraints $P_{u\gamma}\approx 0$ and $\bar P_{u\dot\gamma}\approx 0$ presence. Finally, the latter term in (14.11) including the multipliers

$$\begin{aligned}\{K_{\gamma\dot\gamma}(\vec\sigma), \Psi^{(v)}_{I}(\vec\sigma'')\}_{\text{P.B.}} &= \Big[8(\theta_{\gamma}\bar P_{u\dot\gamma}-\bar\theta_{\dot\gamma}P_{u\gamma}) - iv_{\gamma}\Big((\bar z_{\dot\gamma\dot\delta}+2i\bar\theta_{\dot\gamma}\bar\theta_{\dot\delta})\bar\Psi^{\dot\delta} + (x_{\delta\dot\gamma}+2i\bar\theta_{\dot\gamma}\theta_{\delta})\Psi^{\delta}\Big) + \\ &\qquad + i\bar v_{\dot\gamma}\Big((z_{\gamma\delta}+2i\theta_{\gamma}\theta_{\delta})\Psi^{\delta} + (x_{\gamma\dot\delta}+2i\theta_{\gamma}\bar\theta_{\dot\delta})\bar\Psi^{\dot\delta}\Big)\Big]\delta^{p}(\vec\sigma-\vec\sigma''), \\ \{\Psi^{(v)}_{I}(\vec\sigma''), L^{\alpha}{}_{\beta}(\vec\sigma')\}_{\text{P.B.}} &= iv_{\beta}\Psi^{\alpha}\delta^{p}(\vec\sigma''-\vec\sigma')\end{aligned} \tag{14.15}$$

is also vanished, because of the constraints $\Psi^{\alpha}\approx 0$ and $P_{u\gamma}\approx 0$ presence there.

Conclusion is that the D.B. of $K_{\gamma\dot{\gamma}}$ and $L^{\alpha}{}_{\beta}$ on the surface of the primary constraints coincides with the Poisson bracket and is equal to

$$\{K_{\gamma\dot{\gamma}}(\vec{\sigma}), L^{\alpha}{}_{\beta}(\vec{\sigma}')\}_{\mathrm{D.B.}}|_{\text{constraint surface}} = -\delta^{\alpha}_{\gamma} K_{\beta\dot{\gamma}} \delta^{p}(\vec{\sigma} - \vec{\sigma}')\,. \tag{14.16}$$

Similar analysis can be done for the D.B.'s of the remaining generators of the OSp(1|8) superalgebra and, as a result, one can find that the D.B and P.B realizations of the superalgebra coincide on the primary constraint surface.

In the quantum dynamics the commutator $[\hat{K}_{\gamma\dot{\gamma}}, \hat{L}^{\alpha}{}_{\beta}]$ has to be substituted for the D.B. (14.16) and all the coordinates and momenta should be presented by the correspondent operators. Then the generators of the OSp(1|8) quantum superalgebra include sums of the ordered products of the coordinates and momenta. Moreover, the generators of the OSp(1|8) quantum superalgebra have to be hermitean operators. Following the physical arguments of ref. [42] we choose the $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordering, where $\hat{\mathcal{Q}}$ is a condensed notation for a chain of coordinate operators $\hat{q}$ and, respectively, $\hat{\mathcal{P}}$ for a chain of momentum operators $\hat{p}$. Then, taking into account that any, but fixed $\hat{p}$-operator is not contained two or more times into the $\hat{\mathcal{P}}$-chains, forming the operators $\hat{K}_{\gamma\dot{\gamma}}$ and $\hat{L}^{\alpha}{}_{\beta}$, we find

$$[\hat{K}_{\gamma\dot{\gamma}}(\vec{\sigma}), \hat{L}^{\alpha}{}_{\beta}(\vec{\sigma}')]|_{\text{constraint surface}} = -\delta^{\alpha}_{\gamma} \hat{K}_{\beta\dot{\gamma}} \delta^{p}(\vec{\sigma} - \vec{\sigma}')\,, \tag{14.17}$$

because the operator ordering was not broken during the calculation of the commutator (14.17). Thus, this commutation relation is anomaly free on the primary constraint surface.

The same $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordering procedure was applied to construct the remaining quantum generators of the OSp(1|8) superalgebra and we observed that all (anti)commutators of the superalgebra generators are anomaly free on the surface of some of the primary constraints.

However, this observation is not yet enough to exclude a possibility of quantum anomaly in the total quantum superalgebra of the symmetries of the considered string/brane model. The global OSp(1|8) superalgebra is a subalgebra of the total p- brane superalgebra which includes the gauge subalgebra of the world-volume diffeomorphisms and κ-symmetry transformations formed by the above constructed first-class constraints. Thus, to conclude on the anomaly absence one needs to study the quantum commutation relations of the total quantum superalgebra of the global and world-volume symmetries of the brane model. This problem is under investigation [48].

15. Conclusion

The hamiltonian structure of the simplest twistor $D = 4$ $N = 1$ super p-brane model of which general solution describes the BPS state with exotic fraction of supersymmetry equal to 3/4 was studied here. The covariant division of the brane constraints into the first and second classes was found. As a result, the generators of the local symmetries and the covariant realization of the Dirac matrix $\hat{C}$ were constructed. The matrix $\hat{C}$ was diagonalized and presented in the symplectic form parametrized by the component $\rho^{\tau}(\tau, \vec{\sigma})$ of the brane world-volume vector density $\rho^{\mu}(\tau, \vec{\sigma})$. The corresponding D.B. encoding the hamiltonian symplectic structure of the constrained super p-brane dynamics were constructed.

The D.B. commutation relations between the original p-brane coordinates in the centrally extended twistor superspace were calculated and their D.B. noncommutativity was established. The D.B. noncommutativity in the subspace of the space-time, twistor and TCC coordinates was shown to have a gauge dependent character and can be removed by complete gauge fixing of the exotic κ-symmetry. On the contrary, the D.B. noncommutativity of the Grassmannian θ coordinates among themselves and with the space-time and TCC coordinates was shown to be gauge independent. The constructed Dirac brackets revealed a deep dynamical role of the original auxiliary twistor variables which manifests itself in their noncommutativity with the space-time and TCC coordinates. Using the Dirac brackets we found Dirac bracket realization of the global OSp(1|8) superalgebra. Moreover, the quantum realization of the OSp(1|8) superalgebra was considered applying the $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordering procedure for the superalgebra generators. It was established that this quantum realization of the OSp(1|8) superalgebra does not include anomalous terms on the primary constraint surface. However, we note a possibility of the anomaly presence in the total superalgebra of the brane symmetries unifying the OSp(1|8) superalgebra with the gauge superalgebra associated with the world-volume diffeomorphisms and the Weyl and κ-symmetries. We consider that this problem deserves to be carefully studied and have started it independent investigation using an alternative approach [48]. We have shown there the anomaly presence in the p-brane superalgebra applying the hermitean $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordered representation for the superalgebra generators. It is in agreement with the known result [42], where conformal anomaly in the algebra of tensionless string has been revealed. Now we study the BRST procedure application for the quantization of the considered string/brane model and the construction of its vacuum state and corresponding physical space of states.

Note also, that our analysis is naturally applied for the super p-brane models preserving $\frac{\mathsf{M}-1}{\mathsf{M}}$ fraction of $N = 1$ supersymmetry in higher dimensions. In particular, this concerns the centrally extended $D = 11$ superspace, where 31/32 fraction of $N = 1$ supersymmetry for the studied here model of tensionless super p-brane is preserved [47]. Some additional details arising at the transition from $D = 4$ to $D = 11$ can be found in the recent paper [49], where superstring and super p-brane models preserving fractions of $D = 11$, $N = 1$ supersymmetry less than 31/32 were considered.

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